Milked for All They Are Worth: Livestock Replacement in a Dynamic Discrete Choice Model

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Abstract

Dairy farmers in the United States routinely cull animals before asset replacememt models claim is profit maximizing. This paper examines cow replacement decisions on over 1,000 Wisconsin dairy farms during the period 2011-2014 to discern whether unexpected cow mortality drives replacement decisions. Since animal replacements must be procured ten months in advance in dairy, unexpected asset failure can incur large costs on dairy farms and may encourage early replacement before animal health declines. I model the choice as a dynamic discrete choice problem and estimate the model parameters while taking into account unobserved, fixed cow heterogeneity. Using the conditional choice probability method paired with machine learning, I estimate the cost of mortality at 2,300 USD per death. This is more than twice what is calculated from simulations, which suggests dairy cow replacement models are at odds with producer behavior because they have underestimated the costs of declining animal health. Utilizing farm size heterogeneity, I also find that mortality costs are three times higher on small dairies than on larger ones. Though dairy breeders have prioritized production over health in past decades, these estimates suggest that breeding instead for health and longevity can generate significant cost savings for dairy producers.

Keywords: Dynamic Discrete Choice, Dairy, Asset Replacement, Investment Behavior
JEL Codes: C61, D25, Q12, Q16

1 Introduction

Dairy farms in the United States routinely cull animals before the economics and dairy science literatures claim is optimal. Asset replacement simulations which take into account prices and the management environment suggest keeping cows longer to maximize profit; despite this, dairy farmers have maintained consistently high rates of turnover in their herds by replacing earlier than considered optimal (De Vries, 2013; Van Arendonk, 1988). As a result, dairy cows in the United States on average live to be only about five years old, 25% of their potential lifespan (De Vries, 2013). The consequences of shorter dairy cow lifespan potentially go beyond the farm's bottom line, however. Dairy farms that maintain short cow life also have higher GHG emissions, since more replacement cows must be kept on hand when replacement rates are high (Garnsworthy, 2004; Weiske et al., 2006). While the benefits of keeping cows longer seem clear to scientists, it is not known what motivates dairy farmers to consistently replace earlier. What economic rationale could there be, if any, for such a pattern of asset replacement?

I investigate whether the costs of "unplanned mortality," an event where an animal unexpectedly becomes inoperable and has to be removed from production, account for high replacement rates on Wisconsin dairies. Since dairy cow health declines with age, I explore the extent to which the costs incurred from this type of unexpected exit incentivize dairy farmers to replace cows early as a way of avoiding these costs. To test this hypothesis, I develop and estimate a dynamic discrete choice model of dairy cow replacement using a unique dataset of cow-level production and replacement records on more than 1,000 Wisconsin dairy farms. The empirical model estimates the parameters of the dairy farmer's profit function and is one of the few asset replacement models to incorporate asset-specific heterogeneity into the estimation. Using the conditional choice probability method paired with machine learning, I calculate the costs of unplanned mortality as 2,300 USD per event, 130% higher than what has been previously estimated from simulations. My results suggest that part of the reason dairy farmers do not practice what is considered "optimal" is that previous models have not incorporated or at least underestimated the costs incurred from dairy cows being removed for reasons of poor health. Utilizing farm-size heterogeneity in the dataset, I also find that unplanned mortality costs are almost three times higher on small dairies than on larger dairies. I conclude by calculating producer willingess-to-pay to eliminate mortality in the dataset, finding that on average farms in this sample would pay about 1,900 USD to eliminate unplanned mortality for new cows, 500 USD more than the average cost of a new heifer. While in the past dairy cow genetics have been oriented towards production over health to increase farm profitability, these estimates suggest that breeding instead for animal health and longevity could generate large cost savings for dairy producers, especially smaller dairy farms.

The US dairy sector has been subject to historically low milk prices in the past decade, which has made the question of what sorts of management practices are the most "profitable" more relevant than ever to producers. Many dynamic programming models have been calibrated to calculate the replacement policy that maximizes profit (Arendonk and Dijkhuizen, 1985; Stewart et al., 1977), but these models continue to be odds with what is actually practiced on dairy farms. To understand this discrepancy, I take an empirical approach akin to Rust (1987) and Miranda and Schnitkey (1995) which uses dynamic discrete choice to estimate the cost parameters of the decision directly from observed replacement decisions. This approach is different than the default approach of the dairy science literature which often *prescribes* culling rules for farmers by assuming the costs they face rather than inferring these costs from data (Arendonk and Dijkhuizen, 1985; De Vries, 2013). This paper builds off of Miranda and Schnitkey (1995), the first paper to use dynamic discrete choice to study dairy cow replacement, which could not rationalize the high rates of culling observed on dairies. Their results found that a sizable portion of benefits to culling cattle at early ages accumulated in an intercept term, which they term a "culling premium." As such, the main benefits to culling cattle were unexplained by the states in their model.

I explicitly model one possible cause of replacement to explain their results: unplanned animal mortality. On dairy farms, production downtime is minimized by planning ten months in advance whether a cow will stay in the herd or be replaced by a new animal. This is due to fact that a dairy cow cannot begin producing milk until it has given birth, which it can only do if it is bred ten months before hand. Unplanned mortality is an event where the current cow must exit in the middle of its production cycle, either because of death or a health problem that forces the dairy farm to send the animal to slaughter immediately.[1] This is equivalent to an asset or machine breaking down in the middle of production but, unlike other enterprises, dairy farms are subject to biological constraints that limit their ability to fill this vacancy in the event of a failure. For example, if an animal that was supposed to produce for ten more months dies two months into its cycle, there will be eight months of lost revenue before its replacement will start producing. In addition to this lost revenue, there are high costs of disposal and health treatment that are incurred from having a sick or injured animal; Heikkilä et al. (2012) and De Vries (2013) use dynamic programming simulations and calculate these costs to be in the range of 500-1,000 USD per exit. Facing these costs, dairy farmers may replace cows early to avoid paying the penalty of both lost revenue and disposal costs associated with an animal becoming unexpectedly inoperable.

Unplanned mortality has become even more important to dairy farm decision making given trends in dairy genetics which have favored production over health and longevity. While dairy cows have become more and more productive over the past decades, these improvements have come at the cost of health; dairy cow lifespan has decreased 20% since 1960, in part because of increases in metabolic disease and infertility which forces animals to be removed from the herd earlier and earlier (De Vries, 2017). Given these trends, a conversation has begun in dairy whether these decreases in health in favor of production are truly in the best interest of producers (De Vries, 2013; Knaus, 2009). In addition to potentially harming profitability, shortened dairy cow life may be undesirable from an animal

[1] Example events that are not death but would result in an immediate removal would be an animal sustaining a serious injury, becoming lame, developing an incurable disease, or failing to conceive. These events, together with death, are collectively referred to as "involuntary culling" in the dairy science literature (Fetrow et al., 2006).

welfare perspective and can actually increase GHG emissions from the dairy sector (Oltenacu and Broom, 2010; Weiske et al., 2006). This research provides the first empirical verification of the costs incurred from unplanned mortality as a result of declining health to elucidate the consequences of declining animal health for the profitability of US dairy.

Using the Euler equations in conditional choice probability (ECCP) method, I calculate how costly unplanned mortality is on Wisconsin dairies using empirical replacement decisions while considering a range of motives, including technological progress. I find that the costs of unplanned mortality explain early replacement and vary across herd sizes. My estimate of 2,300 USD is 1,300 USD higher than the upper bound of De Vries (2013) and is even higher on farms with less than 250 cows (3,800 USD). The results are not consistent with technological progress in dairy genetics being a significant factor explaining high rates of replacement, in contrast to what is usually claimed about dairy farm replacement behavior (De Vries, 2017). While asset failure and depreciation is often discussed as a motive in asset replacement (e.g. Burt (1965)), this research demonstrate that this kind of asset failure has a large bearing on replacement behavior in the case of dairy where managers must reduce production downtime by anticipating when assets will be taken out of production. I calculate that dairy farmers would be willing to pay 130 USD on average to insure their newest animals against unplanned mortality, implying they perceive the unplanned mortality "indemnity" for first-year animals at about 1,900 USD (500 USD higher than the price of a new heifer in the same time period). These results suggest that improving animal health and longevity could generate significant cost savings for dairy farmers.

Using farm size heterogeneity, I also calculate the model parameters for different sizes of dairy farm. I find that dairies with less than 250 cows perceive unplanned mortality costs to be as high as 3,800 USD per death, which is nearly three times higher than the cost of unplanned mortality on dairies with more than 500 cows. Since the costs of unplanned mortality are in large part caused by the down time incurred when a replacement is not lined up, these results can be explained by that fact that larger farms can keep more replacement animals on hand and can fill vacancies faster than small dairies. These large discrepancies in cost suggest that breeding for production over health may have the unintended consequence of disproportionately hurting the profitability of small farms, which in turn may further contribute to the consolidation of the dairy industry seen in recent years (Shepel, 2019).

In addition to informing policy, this paper contributes to the literature on asset replacement by flexibly incorporating permanent asset heterogeneity and improving out-of-sample CCP estimation using machine learning. This analysis first improves on previous studies which are only able to control for *across* firm heterogeneity by controlling for *within* firm heterogeneity. Recent advances in CCP methods, specifically the Euler equations in conditional choice probabilities (ECCP) method of Scott (2013) and Aguirregabiria and Magesan (2013), allow estimation of dynamic discrete choice models that can incorporate fixed effects by using discrete analogs of Euler conditions. In my structural model, the Euler condition generates estimates of the cost of unplanned mortality, annual maintenance costs, the parameters of the production function, and also the cow-specific fixed effect. I also improve on previous studies that estimate CCPs with a bin estimator by instead using the random forest algorithm. The random forest algorithm has the advantage of efficiently choosing bins for continuous variables and theoretically improving model efficiency by balancing the in-sample and out-of-sample properties of the probability prediction (Boström, 2008).

The paper proceeds as follows. In Section 2, I review the literature on asset replacement, especially as it relates to replacing dairy cows. I then explain the theoretical model in Section 3 and how I can use data on dairy cow replacement decisions to back out the perceived loss in profit from unplanned mortality using the ECCP method, which I explain in Section 4. Section 5 explains the data used, and Section 6 presents the structural parameters estimated from the data. Section 7 uses these parameters to estimate the willingness-to-pay to eliminate unplanned mortality completely across cow ages and farm types using compensating variation. Section 8 concludes with the policy implications of these results and directions for future research.

2 Literature Review

This paper contributes to a very long tradition in economics of analyzing asset replacement problems. Asset replacement, a special class of the "optimal stopping problem," was analyzed as early as 1849 when German forester Martin Faustmann developed the "Faustmann criterion" for determining the optimal harvest age of a forest (Newman, 2002). With advances in methodology, especially the nested fixed point algorithm of Rust (1987), the economics literature has transitioned away from estimating optimal replacement rules towards using data to estimate the parameters that rationalize observed replacement behavior (Cho, 2011; Rothwell and Rust, 1997; Schiraldi, 2011).

With the exception of Miranda and Schnitkey (1995), studies of dairy cow replacement have not made this transition. While the literature on optimal dairy cow replacement rules is expansive, the majority of studies use simulations to calculate costs rather than empirical models. These models represent the "normative" approach to asset replacement, where the parameters of the problem are assumed, and a dynamic program is solved to recover the optimal culling rule. Attempting to estimate the optimal replacement policy for dairy cattle dates back to Stewart et al. (1977), whose paper in the *Journal of Dairy Science* explicitly modeled and solved the decision using dynamic programming. The state variables included the age of the cow, its body weight, its milk production, and its butterfat production. Subsequent models were more complex and gave less attention to economic incentives and more attention to modeling the underlying biological processes of the dairy cow production system such as milk production (Rogers et al., 1988b; Stewart et al., 1977), fertility (Kalantari et al., 2010; Rogers et al., 1988a) and the incidence of disease (Bar et al., 2008; Heikkilä et al., 2012). Despite the complexity of these models, they often produce prescriptions that are different than the replacement behavior of actual dairy farmers. The majority of these models estimate that 20%-30% of the herd should be culled each year, though the culling rate is usually higher than 30% (De Vries, 2013; Hadley et al., 2006).

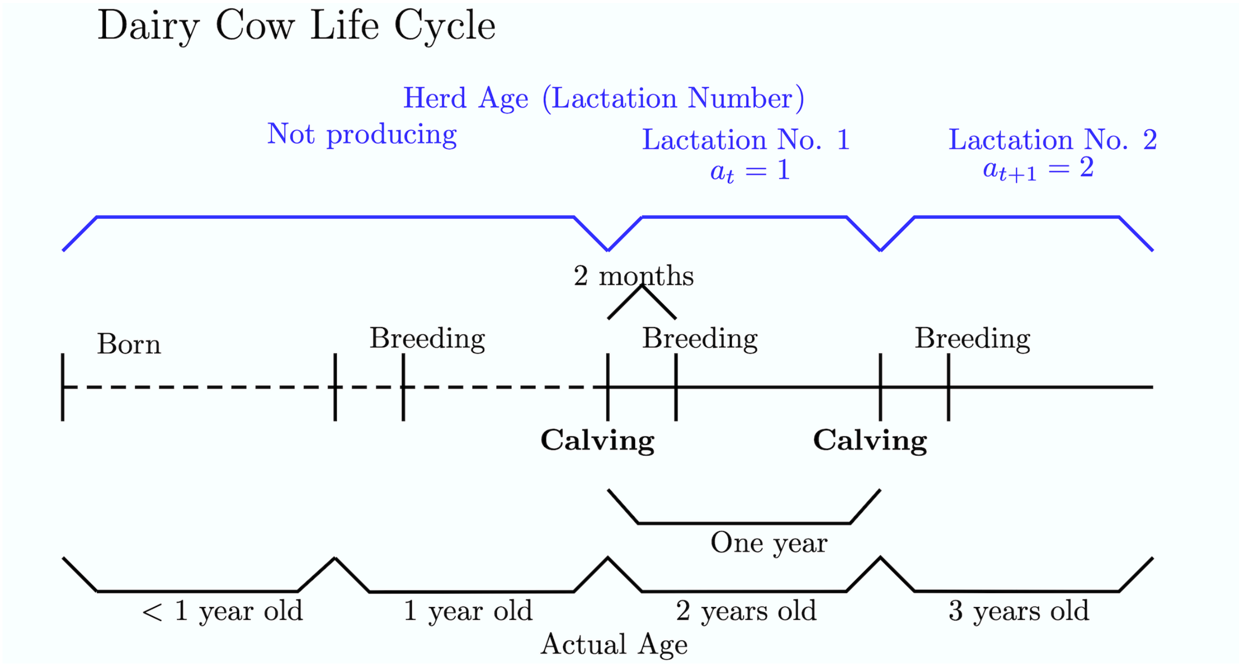
One contribution of this paper is to investigate whether unplanned mortality costs on dairy farms explain this discrepancy. The first and only paper to take this approach to dairy cow replacement is Miranda and Schnitkey (1995), where the authors find that a large component of the gain from replacement is unexplained by their model. They hypothesize that annual costs that are linear in animal age are responsible for early replacement, but for all farms in their study this parameter is statistically insignificant. Instead, the alternative-specific constant for the replacement decision, which is the location parameter of the distribution of the unobserved state ϵ, is large and significant compared to other factors in the model. They theorize that this constant represents factors not explicitly modeled in their profit function, including genetic progress and unseen costs of replacement.

This paper builds on their results by 1) incorporating unplanned mortality as one of their "unseen costs" and 2) controlling for permanent, within firm heterogeneity using the ECCP method. The possibility of asset failure is a salient feature of asset replacement models, especially for replacing dairy cows given how frequent unplanned mortality is (Burt, 1965; Smith, 1973). Asset failure is particularly important in dairy cow replacement because new dairy cows, called "heifers," must be bred ten months before they can be used to replace another cow. The production down time for a dairy farm is minimized when dairy farmers can plan when the current occupant leaves and breed accordingly; unplanned mortality occurs when the current cow leaves ahead of schedule, which will cause lost profit when the stall remains empty until the new cow is ready. In industries where replacements can be procured and put into production immediately asset failure is less likely to affect replacement behavior, but in dairy farming the risk of assets suddenly depreciating is likely to be a large factor in replacement decisions if failure is costly.

In fact, a number of simulations show that the costs of unplanned mortality could be quite substantial for dairy farms. Stott (1994) estimates the costs of infertility using dynamic programming models to help quantify the value of the trait in the selection index; the study arrives at about 25 USD per year per cow as a lower bound and about 100 USD as an upper bound. Heikkilä et al. (2012) also use dynamic programming to calculate the cost of early exit due to mastitis as around 660 USD per exit in Finland. De Vries (2013) estimates the average cost of "involuntary disposal," which includes all of these factors, as 500-1,000 USD per exit in the United States when not considering lost production. These costs are estimated from simulations, however, and not from data. Thus far there is no empirical verification of these costs from actual data, making it impossible to know how costly this kind of health event actually is. By estimating these costs directly from dairy cow replacement decisions, this paper both helps explain the results of Miranda and Schnitkey (1995) and provides an empirical estimate of the total cost of unplanned mortality from the perspective of dairy farmers.

This paper also improves on previous analyses of asset replacement by incorporating permanent, unobserved asset characteristics into the dynamic discrete choice framework via the ECCP method. A threat to identification in these models is the possibility of unobserved asset attributes that can bias parameter estimates, and this is especially so for studying the replacement of animals. In general, the role of within firm heterogeneity in management is usually ignored when studying firm behavior, which is a critical error when studying management of genetic technology (Bloom et al., 2019; Bloom and Van Reenen, 2007). Unlike other industries, each asset on a dairy farm is genetically unique and likely to have idiosyncratic characteristics that would influence decision making. Most asset replacement models, however, assume that assets are homogenous within a firm (Adda and Cooper, 2000; Cooper et al., 1999). Since dairy farmers typically know these genetic traits and the researcher does not, not controlling for these differences in assets can result in biased parameter estimates. To control for this heterogeneity, many papers use mixture distributions which can only control for a pre-specified number of "types" ((Arcidiacono et al., 2016; Arcidiacono and Miller, 2011; Scott, 2013)) Conditional logit models can condition out this heterogeneity, but then cannot incorporate these effects into counter-factual estimation. In this paper, I use the the Euler equation conditional choice probability (ECCP) method which can estimate cow-specific fixed effects and then incorporate these effects into compensating variation

Figure 1



(Aguirregabiria and Magesan, 2013; Scott, 2013). This analysis is a critical step forward in studying the replacement of assets because it is robust to asset heterogeneity that has been ignored in previous analyses and can factor this heterogeneity in to welfare analysis.

To accurately estimate the costs of unplanned mortality from replacement decisions using these methods, I develop a theoretical model that explicitly embeds the risk of mortality in the manager's replacement decision. In the next section, I describe the model and how I recover cost and production function parameters to investigate the causes of replacement from data.

3 Theory Model

In this theoretical model, I examine the case where a dairy farmer maximizes expected current and future profits of a cow stall which at time t contains a cow with age a_t. Throughout the paper, the state a_t refers to the number of producing years the cow has spent in the herd, measured in year long "lactations," as opposed to number of years old (see Figure 1). The cow has an annual production function $y(a_t)$, which is the yield for that entire production year. As production is only a function of age, this model holds all other decisions concerning annual production fixed.[2]

I adopt the standard assumption that the dairy farm operates at full capacity, due to the fact that fixed costs on dairy farms usually make it suboptimal to leave a stall empty (as Miranda and Schnitkey (1995) argues).[3] Formally stated, this model assumes that a manager always fills a stall rather than keep it empty:

Assumption 1. *Fixed extensive decision: the option to leave a stall empty for a year is always dominated by keeping a cow or replacing a cow.*

This implies the manager is faced with a binary decision every year: keep the current cow or buy a replacement with producing age one. Specifically, the manager chooses $i_t \in \{0, 1\}$, where $i_t = 1$ is sell the current cow and buy a replacement and $i_t = 0$ is keep the current cow. Once the decision is made, either the current cow is bred so that it will produce again in ten months or the replacement heifer is bred so that is can replace the current cow in ten months. As Figure 1 shows, this breeding must occur so the dairy cow can begin its next lactation cycle. Since dairy cow lactation cycles are about one year long, this occurs two months into the current cycle in preparation for the next cycle. The farmer, being an expected profit maximizer, considers the difference in expected returns between the current cow a year older and a new cow in its first production year. The expected price of output is p_t and the expected cost of replacement is c_t. If the current animal is replaced, the expected revenue for the next production cycle will be $p_t y(1) - c_t$.

[2]This assumes no variable intensity of use for the asset, which is general standard when studying dairy farming culling behavior (Miranda and Schnitkey, 1995; Smith, 1973). This assumes that there are a fixed number of feed inputs that support milk production, and that the manager views them as fixed throughout the year.

[3]This is due to the fact that fixed costs almost always exceed variable costs on dairy farms. Because of this, it is not often the case that profit margins are so low from milk production that it is more profitable to take a guaranteed loss. In fact, fixed costs can be so high that dairy farms in many cases will respond to drops in price by expanding their herd size rather than increasing it (Atwood and Andersen, 1984).

3.1 The Role of Unplanned Mortality

Without unplanned mortality, the payoff from deciding to continue with the current animal is $p_t y(a_t+1)$. However, it is common for a dairy animal to be to exit unexpectedly either 1) before the annual return is realized (dies while giving birth) or 2) in the middle of its next production cycle. This event is most likely in the first 120 days of their cycle when the animal is weakest, meaning if an animal is removed then most of the year's revenue is not realized. Instead, a new animal must be purchased, meaning the age of the animal regenerates back to one unexpectedly. An animal dying, however, incurs costs that would not have been incurred had the animal been replaced. These costs include the cost of disposing of the carcass, the costs of treating a sick animal that ultimately dies, and lost production.

I model these costs from unplanned mortality as a "penalty," α, which is added to the cost of replacement when the replacement is unplanned.[4] When the animal has to be removed, the next period's return is $p_t y(1) - c_t - \alpha$. The probability that an animal survives to the next period is $S(a_t)$. The payoff function for the decision is thus:

$$R(a_t, p_t, c_t, i_t) = \begin{cases} p_t y(1) - c_t & i_t = 1 \\ S(a_t)\Big(p_t y(a_t+1)\Big) + (1 - S(a_t))\Big(p_t y(1) - c_t - \alpha\Big) & i_t = 0. \end{cases}$$

The manager now has an incentive to replace the animal to avoid paying α. The current period return from replacement, that is $R(a_t, p_t, c_t, i_t = 1) - R(a_t, p_t, c_t i_t = 0)$, would be:

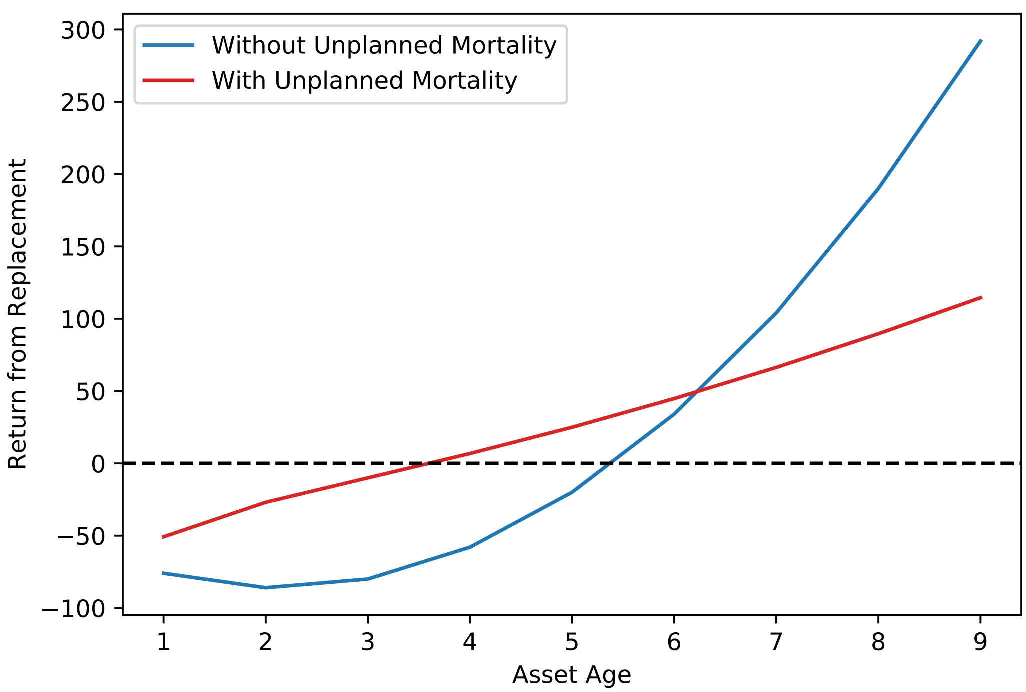
$$(1 - S(a_t))\alpha + S(a_t)\Big(p_t y(1) - p_t y(a_t+1) - c_t\Big). \tag{1}$$

[4]Note that this is independent of production; there are good arguments for making the penalty term proportional to the expected output (some percentage of production is lost). I model it here more simply as independent of production so that the cost can encompass non-production related losses such as disposal costs or health treatment costs.

If $S(a_t) = 1$ for all ages, which is to say that all exit is planned, then α does not affect the decision to replace. Managers replace the current cow when $p_t y(1) - p_t y(a_t + 1) - c_t > 0$, or when the marginal return from replacement exceeds the replacement cost. However, consider the case where the probability $S(a_t)$ is decreasing in age (intuitively, older cows are more likely to have to be removed), or at least decreasing after some point. As age progresses, α will get larger and the previous criterion will get smaller; intuitively, the growing risk of the cow exiting prematurely over time eclipses any marginal return from keeping the current cow.

As an illustration, consider the parametric example in Figure 2 using a quadratic, concave functional form for $y(a_t)$. The survival probability is modeled as a variant of the Weibull hazard rate function and is monotonically decreasing as age increases. The payoffs with and without asset failure are graphed in red and blue.

Figure 2: Payoffs



State Values		Parameter Values		Survival Function
p	10	(β_1, β_2)	(5, -0.8)	$S(a_t) = 0.8a^{-0.5}$
c	50	α	50	

The blue line shows that under no unplanned mortality the optimal policy is to replace at about age five, about two years after the production function is maximized ($\beta_1/2\beta_2 = 3.125$). However, with the penalty, the optimal replacement age is two years younger, at about three, because the risk of incurring mortality cost is too high. The only case when the assets will be replaced at the same time regardless of output price or replacement cost is when $\alpha = 0$. The payoff with unplanned mortality also has a smaller slope with respect to age; this is due to the effect of $S(a_t)$ on the other states, which decrease in importance as age increases. Because of this difference in curvature, even small increases in α will cause a large discrepancy between the optimal culling rules.

This simple example illustrates why a model that does not incorporate unplanned mortality will have a large, positive "culling premium." If, as in the case of Miranda and Schnitkey (1995), we were to estimate model where $S(a_t) = 1$ for all ages, there would need to be an intercept adjustment for the discrepancy between the curves in Figure 2. Thus, including the penalty α in the model can potentially explain the culling premium found in Miranda and Schnitkey (1995).

In addition to unplanned mortality, there are other motives for replacing dairy cows at young ages. Three of these motives that I include in the model are maintenance cost, asset performance, and technological progress. Miranda and Schnitkey (1995) claim that maintenance costs of aging cattle that are linear in age can explain early replacement. If these costs are included, age affects the current-period payoff linearly through maintenance cost and also non-linearly through the transition probability $S(a_t)$. To compare the Miranda and Schnitkey (1995) model to the one I develop here, I include the same linear maintenance cost function $M(a_t) = \gamma a_t$ in the payoff. Another motive for early replacement is observed asset performance, which I model this by including an additional state: the production shock η_t. This state is the deviation from the asset's expected performance, $y(a_t)$, which captures other aspects of productivity not explained by age. This state, like age, is influenced by the choice i_t. When the asset is not replaced, the next cycle's shock η_t is drawn from

$\eta_t \sim N(\rho\eta_{t-1}, \sigma_\eta)$, where ρ is an autocorrelation coefficient. When the asset is replaced, η is expected to be zero, or $\eta_t \sim N(0, \sigma_\eta)$. Since shocks are correlated across time, relatively poor performance in one period may cause a cow to be replaced to stop the shock from carrying over to the next period.

Finally, technological progress in dairy cow genetics may provide an incentive to replace old cows with new ones since newer dairy cows may be more productive. This is often a key feature of asset replacement models in industries where asset performance is improving over time (Bethuyne, 1998; Cho, 2011; Perrin, 1972). When each new generation of asset is more productive, there may be a large opportunity cost to using the current asset instead of replacing it with a new one. In the case of dairy, genetic improvement has caused dairy cow milk yield to be almost linearly increasing over the past fifty year (Thornton, 2010). For this reason, genetic improvement has been the most cited explanation for why dairy farmers have an incentive to replace cows early (De Vries, 2017; Miranda and Schnitkey, 1995; Smith, 1973). I model this by including a time trend in the payoff for replacement, which allows the payoff from replacement to grow linearly over time. This trend is expected to be positive and significant if expectations of technological progress play a role in culling decisions.

3.2 The Dynamic Model

The manager's full, dynamic decision problem is represented by an infinite-horizon, dynamic program, with discount rate $\delta \in [0, 1)$. The Bellman equation for the optimization problem is:

$$V(x_{jt}, z_t) = \max_{i_{jt} \in \{0,1\}} R(x_{jt}, z_t, i_{jt}) + \epsilon(i_{jt}) + \delta E(V(x_{j,t+1}, z_{t+1}) | x_{jt}, z_t, i_{jt}) \tag{2}$$

where $x_{jt} \equiv (a_{jt}, \eta_{jt})$ and $z_t \equiv (p_t, c_t)$ are cow-level states and market-level states respectively for cow j in period t.[5] In addition to including the value function V in the payoff, there

[5] Assuming that the prices p_t and c_t are exogenous is equivalent to assuming that dairy farmers are price takers. This is generally true for dairy farms, especially dairy farms in Wisconsin, where very few farms keep

is an additional state ϵ that represents the influence of states not observed in the data. In what follows, I assume:

Assumption 2. *Conditional independence: the transition of states x and z are conditionally independent of ϵ.*

Assumption 3. *Additively separable type 1 extreme value: the error ϵ is additively separable in the payoff and is distributed type 1 extreme value.*

Assumption 2 is common in models of this type (e.g. (Rust, 1987)). Hotz and Miller (1993) argue that ϵ satisfies conditional independence by construction if R represents an ex-ante payoff,and so ϵ is unobserved, expectational noise. The power of this assumption is that it frees us from having to take an integral over V with respect to ϵ, instead integrating only over the states x_{jt} and z_t. Assumption 3 implies that the probabilities have a logit form, which permits the derivation of a closed form expression for estimating the model parameters. It also implies a closed-form expression for the differences in value functions, which is detailed in the next section.

The transition functions for the exogenous states p_t and c_t are modeled as normally distributed random variables that are AR(1). The shock distribution is similarly modeled as normally distributed with variance σ_η^2. For the transition of age, a_t always transition to 1 if $i_{jt} = 1$, but otherwise returns to 1 with probability $1 - S(a_{jt})$ and transitions to $a_{jt} + 1$ with probability $S(a_{jt})$. This implies that the continuation value when $i_{jt} = 0$ is a weighted combination of $\bar{V}_1(x_{jt}, z_t) = E(V(x_{j,t+1}, z_{t+1})|x_{jt}, z_t, i_t = 1)$ and $\bar{V}_0(x_{jt}, z_t) = E(V(x_{j,t+1}, z_{t+1})|x_{jt}, z_t, i_{jt} = 0)$. When entering the "unplanned mortality" state of nature, the value function evolves as if a new asset was purchased.

Taking shocks and maintenance cost into account, the payoff function can be rewritten as:

more than 200 cows, so market power is very dispersed.

$$R(x_{jt}, z_t, i_t) = \begin{cases} \tau t + p_t y(1) - M(1) - c_t & i_{jt} = 1 \\ S(a_{jt})\Big(p_t y(a_{jt}+1) + \rho\eta_{jt}p_t - M(a_{jt}+1)\Big) + & i_{jt} = 0. \\ (1 - S(a_{jt}))\Big(p_t y(1) + \rho\eta_{jt}p_t - M(1) - c_t - \alpha\Big) & \end{cases}$$

$$\text{where} \quad M(a_t) = \gamma a_t, \quad y(a_{jt}) = \beta_0 + \beta_1 a_{jt} + \beta_2 a_{jt}^2, \quad \{x_{jt}, z_t\} = \{a_{jt}, \eta_{jt}, p_t, c_t\}$$

I now include the effect of maintenance costs, production shocks, and a time trend τ. The shock η_{jt} always affects the payoff when $i_{jt} = 0$; this is to take into account the fact that asset failure can have repercussions related to the previous cycle's performance.[6] Using the shorthand $S_{jt} = S(a_{jt})$, I now write the difference in current-period payoffs as:

$$\begin{aligned} R(x_{jt}, z_t, i_{jt} = 1) - R(x_{jt}, z_t, i_{jt} = 0) =& \mu + \tau t + \alpha(1 - S_{jt}) - \rho\eta_{jt}p_t - S_{jt}c_t + \gamma S_{jt}a_{jt} \\ & - (\beta_1 + 2\beta_2)S_{jt}a_{jt}p_t - \beta_2 S_{jt}a_{jt}^2 p_t \qquad (3) \\ R(x_{jt}, z_t, i_{jt} = 1) - R(x_{jt}, i_{jt} = 0) =& \theta X \end{aligned}$$

where $\theta = \Big(\mu, \tau, \alpha, -\rho, -1, -\gamma, -(\beta_1 + 2\beta_2), -\beta_2\Big)$ is the parameter vector and $X = \Big(1, t, 1 - S_{jt}, \eta_{jt}p_t, S_{jt}c_t, S_{jt}a_{jt}, S_{jt}a_{jt}p_t, S_{jt}a_{jt}^2 p_t)$ is the data matrix. Here μ is the difference in means between $\epsilon(1)$ and $\epsilon(0)$, which is the "culling premium" from Miranda and Schnitkey (1995). It contains benefits to choosing to replace that are unexplained by the other factors in the model. Knowing the current period payoff, the next step is to take into account the effect that today's replacement has on the future stream of profits. In the next section, I use the inversion theorem of Hotz and Miller (1993) to derive the difference in value functions as a function of replacement probabilities and derive an estimating equation using derivations in

[6] To make the shock transmit only in the case of survival, we need only multiply the term $\eta_{jt}p_t$ by the survival rate S_{jt} in the regression equation that follows.

Table 1: Model Summary

Endogenous States (x_{jt})	a_{jt}	Age
	η_{jt}	Production shock
Exogenous States (z_t)	p_t	Output price
	c_t	Replacement cost
Controls	$i_{jt} \in \{0, 1\}$	Replacement decision
Technology	$y(a_{jt}) = \beta_0 + \beta_1 a_{jt} + \beta_2 a_{jt}^2$	Total milk output at age a_{jt}.
	$M(a_{jt}) = \gamma a_{jt}$	Maintenance cost function.
	$S(a_{jt})$	Survival rate
	$P(a_{j,t+1} = 1 \mid i_{jt}) = \begin{cases} 1 & i_{jt} = 1 \\ 1 - S(a_{jt}) & i_{jt} = 0 \end{cases}$	Evolution of a_{jt}
	$P(a_{j,t+1} = a_{jt} + 1 \mid i_{jt}) = \begin{cases} 0 & i_{jt} = 1 \\ S(a_{jt}) & i_{jt} = 0 \end{cases}$	
Payoff, $R(x_{jt}, z_t)$	$\mu + \tau t + p_t y(1) - \gamma - c_t$	If $i_{jt} = 1$
	$S(a_{jt})\Big(p_t y(a_{jt} + 1) + \rho \eta_{jt} p_t - \gamma(a_{jt} + 1)\Big) +$ $(1 - S(a_{jt}))\Big(p_t y(1) + \rho \eta_{jt} p_t - \gamma - c_t - \alpha\Big)$	If $i_{jt} = 0$
Parameters	β_1, β_2, γ	Production and cost parameters
	$\delta \in [0, 1)$	Discount factor
	ρ	Shock correlation
	τ	Time trend
	α	Cost of unplanned mortality
	μ, λ	Location and scale of error term

Aguirregabiria and Magesan (2013) and Scott (2013).

4 Methodology

When $\delta > 0$, the decision to replace also takes into account the effect that replacement has on future decisions, which is

$$\Delta V(x_{jt}, z_t) = E(V(x_{j,t+1}, z_{t+1})|x_{jt}, z_t, 1) - E(V(x_{j,t+1}, z_t)|x_{jt}, z_t, 0).$$

Rust (1987) develops an approach for estimating this factor, which is to solve a value function iteration problem to find $V^*(x_{jt}, z_t)$ across all states, calculate $\Delta V(x_{jt}, z_t)$, and include it in the maximum likelihood estimation. The problem is computationally burdensome since $\Delta V(x_{jt}, z_t)$ is also a function of parameters θ, so in any optimization routine the value function iteration must be done for each new likelihood calculation for a candidate estimate of θ.

Instead of using Rust's nested fixed-point method, I use the CCP estimator derived by Hotz and Miller (1993) and extended by Arcidiacono and Miller (2011). Define the probability of taking action k conditional on endogenous states x_{jt} and exogenous states z_t as the "conditional choice probability," $P_k(x_{jt}, z_t)$. Also denote the transition probabilities for x and z as f_x and f_z.

Finally, define the "conditional value function," the payoff from choosing action i and acting optimally from then on, as $v(x_{jt}, z_t, i_{jt})$. Using this notation, their recursive relationship is given by:

$$v(x_{jt}, z_t, i_{jt}) = R(x_{jt}, z_t, i_{jt}) + \delta E(\bar{V}(x_{j,t+1}, z_{t+1})|x_{jt}, z_t, i_{jt}) \tag{4}$$

where $\bar{V}$ is the "ex ante" or "unconditional" value function where every decision after the one in the current period is made optimally and so does not depend on i_{jt}.

According to Lemma 1 of Arcidiacono and Miller (2011), there is a function ψ such that $\psi(x_{jt}, z_t, i_t) = \bar{V}(x_{jt}, z_t) - v(x_{jt}, z_t, i_{jt})$. Now, using the function ψ we can substitute $\bar{V}$ into Equation 4:

$$v(x_{jt}, z_t, i_{jt}) = R(x_{jt}, z_t, i_{jt}) + \delta E(v(x_{j,t+1}, z_{t+1}, k) + \psi(x_{j,t+1}, z_{t+1}, k)|x_{jt}, z_t, i_{jt})$$

where k is an arbitrary choice. The reason k can be any given choice is that the term ψ will essentially "penalize" the returns if this is not the optimal action (Arcidiacono and Miller, 2011; Hotz and Miller, 1993).

Hotz and Miller (1993) show that Assumption 3 implies that $\psi(x_{jt}, z_t, k) = .577 - ln(P_k(x_{jt}))$ (.577 being Euler's constant), where $P_k(x_{jt})$ is the CCP of taking action k. The most useful choice of k is $k = 1$, which assumes that all cows are replaced next period, in order to exploit the principle of "limited dependence." Limited dependence is a special feature of models that involve a "renewal decision," which is a decision that resets one of the states so that previous actions have no further effect on the future. In this case, replacing a cow renews the state a_{jt} back to 1, and if the cow is replaced at $t + 1$ then there will always be a new cow at $t + 2$ that is unaffected by decisions in t.[7]

If $k = 1$, then the difference in value functions $v(x_{jt}, z_t, i_{jt} = 1) - v(x_{jt}, z_t, i_{jt} = 0)$ is only a function of the payoffs in period t and $t + 1$, since the decision is identical from $t + 2$ onward. The resulting difference in value functions is

[7] See the example of Aguirregabiria and Magesan (2013) for a specific application to dairy cattle replacement.

$$
\begin{aligned}
v(x_{jt}, z_t, 1) - v(x_{jt}, z_t, 0) =& R(x_{jt}, z_t, 1) - R(x_{jt}, z_t, 0) \\
&+ \delta\Big(E(R(x_{j,t+1}, z_{t+1}, 1) - \psi(x_{j,t+1}, z_{t+1}, 1)|x_{jt}, 1) \\
&- E(R(x_{j,t+1}, z_{t+1}, 1) - \psi(x_{j,t+1}, z_{t+1}, 1)|x_{jt}, 0)\Big) \\
=& R(x_{jt}, z_t, 1) - R(x_{jt}, z_t, 0) \\
&+ \delta \sum_{z_{t+1}=1}^{Z} \sum_{x_{j,t+1}=1}^{X} \Big(R(x_{j,t+1}, z_{t+1}, 1) - \psi(x_{j,t+1}, z_{t+1}, 1)\Big) \\
&\Big(f_x(x_{j,t+1}|x_{jt}, 1) - f_x(x_{j,t+1}|x_{jt}, 0)\Big) f_z(z_{t+1}|z_t). \qquad (5)
\end{aligned}
$$

Recalling that $\psi(x_{jt}, z_t, k) = .577 - ln(P_k(x_{jt}))$, this reduces to:

$$
\begin{aligned}
v(x_{jt}, z_t, 1) - v(x_{jt}, z_t, 0) =& R(x_{jt}, z_t, 1) - R(x_{jt}, z_t, 0) \\
&+ \delta \sum_{x_{t+1}=1}^{X} \sum_{z_{t+1}=1}^{Z} \Big(R(x_{t+1}, z_{t+1}, 1) + lnP_1(x_{t+1}, z_{t+1})\Big) \\
&\Big(f_x(x_{t+1}|x_{jt}, 1) - f_x(x_{t+1}|x_{jt}, 0)\Big) f_z(z_{t+1}|z_t),
\end{aligned}
$$

after noting that I can factor out f_z because, being comprised of only exogenous states, it is not affected by the decision i_{jt}.[8] To calculate the relative payoff from replacing, given by $v(x_{jt}, z_t, 1) - v(x_{jt}, z_t, 0)$, I need only the CCPs across different states, $P_1(x_{jt}, z_t)$, and the difference in transition probabilities, $f(x_{t+1}|x_{jt}, 1) - f(x_{t+1}|x_{jt}, 0)$. To have identification, I also normalize one payoff to zero (Magnac and Thesmar, 2002). In this case, I choose to normalize the payoff from replacement to zero, which is equivalent to subtracting $R(x_{jt}, z_t, 1)$ from both payoffs. Recalling that $S(a_{jt}) = f(a_{jt} + 1|a_{jt}, 0)$ and $1 - S(a_{jt}) = f(1|a_{jt}, 0)$, the

[8] Also note that it is now easier to see why ψ "penalizes" the payoff when $P_1 \neq 1$; if $P_1 < 1$, then $\psi < 0$, but the payoff is unchanged if $P_1 = 1$.

final expression for $\Delta V(x_{jt,z_t})$ is

$$\Delta V = FV_1 + S(a_t)FV_2 \quad \text{where} \tag{6}$$

$$\begin{aligned}
FV_1 = \sum_{z_{t+1}=1}^{Z} \sum_{\eta_{t+1}=1}^{E} & \Big(lnP_1(1, \eta_{t+1}, z_{t+1}) \Big) \\
& \Big(f_\eta(\eta_{t+1}|\eta_t, 1) - f_\eta(\eta_{t+1}|\eta_t, 0) \Big) f_z(z_{t+1}|z_t) \\
FV_2 = \sum_{z_{t+1}=1}^{Z} \sum_{\eta_{t+1}=1}^{E} & \Big(lnP_1(1, \eta_{t+1}, z_{t+1}) - lnP_1(a_t + 1, \eta_{j,t+1}, z_{t+1}) \Big) \\
& \Big(f(\eta_{t+1}|\eta_t, 0) \Big) f_z(z_{t+1}|z_t)
\end{aligned}$$

(see Appendix A for derivation).

Using a first-stage estimate of P_1, I now include FV_1 and $S(a_t)FV_2$ as two additional regressors in the model to proxy for the continuation value after estimating the transition probabilities f_η and f_z.

4.1 First-Stage Estimation

The above is a two-step estimator: first calculate the CCP $\hat{P}$, and then estimate the regression equation. The first step, however, requires calculation of $\hat{P}$, both in-sample and out-of-sample. Unfortunately, I need to observe all combinations of ages, production shocks, and prices to have accurate estimates of P_1 across all states. A common way to estimate P_1 is to use some kind of bin estimator (Scott, 2013) or a logit model with several combinations of the state variables used as predictors (Arcidiacono and Miller, 2011). The first method requires making judgements on the size of the bins, which can be difficult when states are fully continuous (as in my case here with output price p_t and replacement cost c_t). The second method does not require choosing bins, but tends to need many combinations of state variables to fit well in-sample, which makes the out-of-sample fit worse due to over-fitting.

I choose to predict P_1 using a random forest algorithm as a compromise between these two methods for the following two reasons. First, a random forest model prevents the econometrician from having to choose bins, as it essentially selects the bins using cross-validation. Many of the hyper parameters in a random forest, such as the number of leaves or the minimum sample on a leaf, are essentially changing the bin size. It is therefore a more sophisticated bin estimator that frees the econometrician from having to choose bins for continuous variables. When a random forest model is trained using a Brier-score loss function, it delivers the desired probabilities of replacement rather than doing classification (Boström, 2008).

Second, using a method with cross-validation prevents the model from over-fitting and causing poor out-of-sample performance. A logit model with many combinations and polynomial expansions of state variables, as recommended by Arcidiacono and Miller (2011), is a classic example of a model that will over-fit; it will produce accurate in-sample probabilities but will do poorly at predicting combinations of ages, shocks, and prices that are not seen in the data. This will produce inaccurate estimates of ΔV, in particular. To address this, I deploy a machine learning approach that will correct for overfitting.

I also need to estimate the transition probabilities f_z and f_η in the first stage using an AR(1) regression where the error is normally distributed. I use the same regression to find flow probabilities for production shock η, though here the data for η comes from data on the animal's milk production. Specifically, I use the milk production model in Kearney et al. (2004) to predict milk yield for a given animal; the residual for each lactation is my estimate for η. This proxies for the production shock in the structural model because the production of the animal is net of any observable predictor of milk production on the farm. Since the milk production model uses herd fixed effects, the residual is actually its deviation from the herd average. I believe this is a good approximation of a "deviation" from its expected return from the perspective of the manager. More information about the milk production model is given in Appendix B.

Finally, the survival probability S_{jt} is estimated from the data based on the percentage of cows at each age that exit the herd in the first 120 days of their lactation. Due to age being a discrete variable, there is no reason for any parametric assumption. The literature on dairy cow culling calculates the probability of "involuntary exit" for each age (see, for example, Stott (1994) and Van Arendonk (1985)). In this particular application, I assume that the shape of S_{jt} is exogenously imposed. While it is known that the manager's actions can have an effect on the rates of exit, it is not clear from that dairy farm managers actually treat this as a choice variable. Management actions that have an effect on cow death and infertility are broad structural changes that cannot be changed in the short run. We can allow for *shifts* in the level of S_{jt} at all ages using fixed effects, while still assuming that the decrease in S_{jt} between ages is common to all animals.

4.2 Second-Stage Estimation

To estimate θ, I assume a value for the parameter δ to give enough degrees of freedom (see Arcidiacono and Miller (2011) for an explanation of when δ is identified). Given an estimate of the survival probability S_{jt}, I have the following reduced form logit model that maps to the structural coefficients:

$$P(i_{jt} = 1 | x_{jt}, z_{jt}) = \frac{e^{\lambda\theta X + \delta\Delta V}}{1 + e^{\lambda\theta X + \delta\Delta V}}. \tag{7}$$

The parameter λ is the scale parameter of the distribution, and is identified as the coefficient on the term $S_{jt}c_t$ because its coefficient in the structural model is -1. In order to interpret the coefficients in dollar terms, I divide through by λ. The reduced form coefficients, after dividing through by λ, are

$$\begin{aligned} X =& \Big(1, 1 - S_{jt}, \eta_{jt}p_t, S_{jt}c_t, S_{jt}a_{jt}, S_{jt}a_{jt}p_t, S_{jt}a_{jt}^2 p_t\Big) \\ \theta/\lambda =& \Big(\mu_j, \alpha, -\rho, -1, \gamma, -(\beta_1 + 2\beta_2), -\beta_2\Big) \end{aligned}$$

$$\theta_1 = \mu \qquad \theta_2 = \alpha \qquad \theta_3 = -\rho$$
$$\theta_5 = \gamma \quad \theta_6 = -\beta_1 - 2\beta_2 \quad \theta_7 = -\beta_2$$

So now I can recover the structural parameters:

$$\mu = \theta_1 \qquad \alpha = \theta_2 \qquad \rho = -\theta_3$$
$$\gamma = \theta_5 \quad \beta_1 = \theta_6 - 2\theta_7 \quad \beta_2 = -\theta_7$$

Note that here θ_1 is essentially estimating the "willingness to pay" for a lower mortality rate, $1 - S_{jt}$. In this structural model, this is equal to the cost of mortality, α.

In contrast to previous work, specifically that of Miranda and Schnitkey (1995) and Aguirregabiria and Magesan (2013), I do not estimate the parameters of the production function from outside the structural model. Instead, the production function parameters β_1 and β_2 are identified off of interactions between age, survival rate, and the output price. Were the parameters to be estimated with milk production data and then plugged into the model, this would be assuming that the econometric estimates are the parameters the manager assumes. Unfortunately, this ignores the fact that a cow's milk production curve may be perceived differently by the manager than what could be discovered from a regression. For example, the farmer may have information about the cow's production curve under their own management that would not be uncovered with an econometric regression. The manager may also have a different notion of when an animal's milk production is maximized. This approach allows any of these possibilities to be true but changes the interpretation of β_1 and β_2: they are no longer the parameters of the "empirical" production function, but rather the parameters of the "perceived" production function from the perspective of the manager. To understand the "perceived" production function, I also calculate the age of maximum production $a^* = -\frac{\beta_1}{2\beta_2} = \frac{\theta_5 - 2\theta_6}{\theta_6}$. If a^* differs from what is estimated from empirical models, this is evidence than manager's expect a different production function than the one calculated from dairy cow testing data.

Estimating θ using maximum likelihood, unfortunately, cannot incorporate permanent, unobserved asset heterogeneity without conditioning out the cow effects altogether. While this would control for cow genetics when estimating θ, it does not allow calculation of marginal effects or counterfactual calculations with these genetic effects. To overcome these limitations, I use the ECCP method which can flexibly incorporate fixed effects in the estimation equation. This method has been implemented using GMM (Aguirregabiria and Magesan, 2013) and OLS (Scott, 2013) by utilizing moment conditions. Taking logs of both sides of Equation 7 gives the following moment condition:

$$X\theta - \frac{1}{\lambda}(\delta\Delta V + \Delta\nu) = 0, \tag{8}$$

where $\Delta\nu = ln\left(\frac{P(i_t=1|x_{jt},z_t)}{P(i_t=0|x_{jt},z_t)}\right)$. Scott (2013) rearranges the above moment condition to get a regression equation:

$$\begin{aligned} X\theta - \frac{1}{\lambda}(\delta\Delta V + \Delta\nu) &= 0 \\ \delta\Delta V + \Delta\nu &= \lambda X\theta \\ \delta\Delta V + \Delta\nu &= \lambda X\theta + \tilde{\xi} \\ \tilde{Y} &= \lambda\theta X + \tilde{\xi} \\ \text{s.t.} \quad \tilde{Y} &= \delta\Delta V + \Delta\nu. \end{aligned} \tag{9}$$

The regression equation is given by the reduced form expression:

$$\begin{aligned} \tilde{Y}_{jt} =& \mu_j + \tau t + \alpha(1 - S(a_{jt})) - \rho\eta_{jt}p_t - S(a_{jt})c_t + \\ & \gamma S(a_{jt})a_{jt} - (\beta_1 + 2\beta_2)S(a_{jt})a_{jt}p_t - \beta_2 S(a_{jt})a_{jt}^2 p_t + \tilde{\xi}_{jt}. \end{aligned}$$

Note that with this method I do not have to condition out fixed effects, and so obtain estimates of μ_j for every animal in the dataset. I can can use Equation 7 and our estimate of θ to construct counterfactual estimation of probabilities without assuming the fixed effect is zero.

Unobserved benefits to replacement that are time variant now manifest in a regression error, $\tilde{\xi}$. Scott (2013) argues that $\tilde{\xi}$ is actually a compound error term, one part expectational error and one part "unobservable shock"; the first of these is arguably uncorrelated with the information known to the manager, instead having to do with the evolution of exogenous market variables. The second component is likely not exogenous to payoffs, however, and so needs to be addressed.

4.3 Endogeneity and Identification

There are two main concerns for endogeneity when estimating θ: herd environment and unobserved health information. Thomsen and Houe (2006) show that herd environment affects the survival rate; more intensive dairies, for example, generally have higher rates of exit. Since management intensity (e.g., number of times milked or type of feed used) is not in the model, this can confound estimates of α and other parameters. I assume that cows can have different levels of survival rate but that their survival rate as a function of age has a common curvature:

Assumption 4. *Exogenous survival rate curvature: the curvature of the survival rate S with respect to age is fixed from the perspective of the manager and is the same across all farms.*

I assume that heterogeneity in the perception of survival rate across farms comes only from linear shifts in the curve at the farm level. This means that farms can have differing survival rates provided they are linear shifts in the rate *at every age*. There is no evidence one way or the other in the literature about how dairy farmers perceive the survival rate, but it seems likely that dairy farmers would not have radically different ideas of how survival decreases with age.

Unobserved health information is also a source of endogeneity that manifests at the animal level. A farmer may observe something about one animal that updates the probability of survival while at the same time affecting replacement. There is no explicit "health state" in the model that captures this information, meaning it would instead show up in ϵ. If the health states are fixed across the animal's lifestyle, we can use cow-level fixed effects to control for this confounding factor. This will omit all cows that do not survive past their first lactation, which is appropriate given first year cows being culled are a special case of culling. These cows are usually sold in their first lactation, alive, to generate income. This sort of culling is not the focus of this analysis, as it is not culling initiated with a replacement in mind. In the analysis that follows, I default to cow-level fixed effects.

Finally, neither of the above fixed-effects methods deal with health shocks, which would affect both survival and replacement. One example is the onset of a disease that is unobserved in the data but observed by the manager. One proxy measure of such an event found in the data is somatic cell count (SCC), which is a measure of the bacteria count in the milk. SCC is usually monitored closely by managers because a high SCC indicates the onset of mastitis, the most common lactating dairy cow disease. SCC is available from the data but is not currently modeled here; arguably this measure produces just as many endogeneity problems because SCC is also correlated with certain management practices. For now, I use SCC to check the model results for robustness by including it as a variable in the regression. This checks the predictive power of SCC in the model and also the effect of including it on other parameter estimates. More information on robustness checks are available in the Appendix C.

5 Data

To estimate the model parameters, I use cow-testing records from Dairy Herd Improvement (DHI) herds in Wisconsin served by one Dairy Records Processing Center representing 90% of the DHI herds in the state; DHI herds represent roughly half of total dairy farms in Wisconsin. DHI Associations are producer cooperatives established in the early 20th century

U.S. to collect information on dairy cow production for benchmarking of individual cows and whole herds. Herds that are a member of DHI have records collected every month on their cows' milk production, including fat and protein yield, as well as somatic cell count (SCC) and breeding decisions. All cows in a given milking herd have records collected monthly, so when a cow exits the milking herd it also exits the data.

The data covers the period from June 2011 to January 2015. It includes data on roughly 150,000 cows across 1,285 herds, totalling 355,734 lactation records. I look specifically at lactation-level records, which record the total fat and protein during the lactation. The raw data contains many more herds than the 1,300 included in my sample, but these were dropped from the analysis based on three criteria. One, herds had to have at least 40 milking cows at any given point in the data. Two, herds had to have been observed from June 2011 up until December 2014 (making a balanced panel). Three, herds could not have large fluctuations in herd size; I dropped herds whose herd size had a coefficient of variation of more than one.[9] For animal-level records, lactations above five or six are routinely omitted (Pinedo et al., 2014; Weigel et al., 2003) because of survival bias; animals that live to be that long are usually extraordinarily good at producing milk and do not represent a typical cow. Including these records would also cause issues with studying replacement, because the rate of culling for those animals is usually either zero or one. I include animals up to lactation eight, which cuts out only about 1% of the data.

5.1 Exit Rates

A dairy cow "exit" occurs when a dairy cow leaves the dataset. In this data, if a dairy cow leaves the dataset less than six months before the end of the sample time frame, the observation is considered to be right censored rather than an exit. Figure 4 shows the rate of exit for cows that are uncensored on average and also across herd size. No cows that exit

[9]Herds have wildly fluctuating herd size when there is an issue collecting data on all the cows, so these herds are omitted. A certain amount of leeway is allowed in the herd size because herd size can fluctuate even when dairy farms are not actively scaling up or down. Herd size can fluctuate temporarily, for example,

	Record Counts
Herds	1,285
Cows	151,722
Cows (not Right Censored)	60,248
Lactation Records	355,734

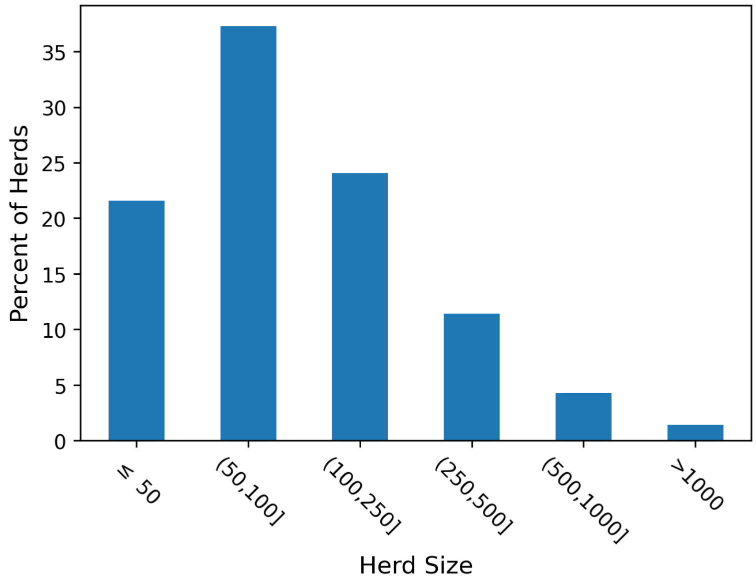


Figure 3: Record Counts and Herd Sizes

the data at their first lactation are considered in this analysis, and so their exit rate at the first lactation is zero. The exit rate jumps to about 40% in their next lactation, however. While dairy cow milk output is maximized at about lactation three or four, only about half of the cows make it to this age. Across herd size, there are not significant differences in exit rates, although a slightly smaller percentage of cows exit at the first lactation on smaller farms.

In this dataset, we also know whether or not a dairy cow was bred. Breeding is an important decision to observe because it also determines whether or not a cow produces next year. If the cow is not bred, then it will eventually leave the milking herd when its production cycle ends. Figure 4 shows the rate at which cows are bred. While exit rates are fairly homogenous across dairy farm size, breeding rates are slightly more heterogeneous across herd size. For larger farms, a slightly higher percentage breed more of their cos in the

because a replacement is being purchased or is not quite ready from the replacement herd.

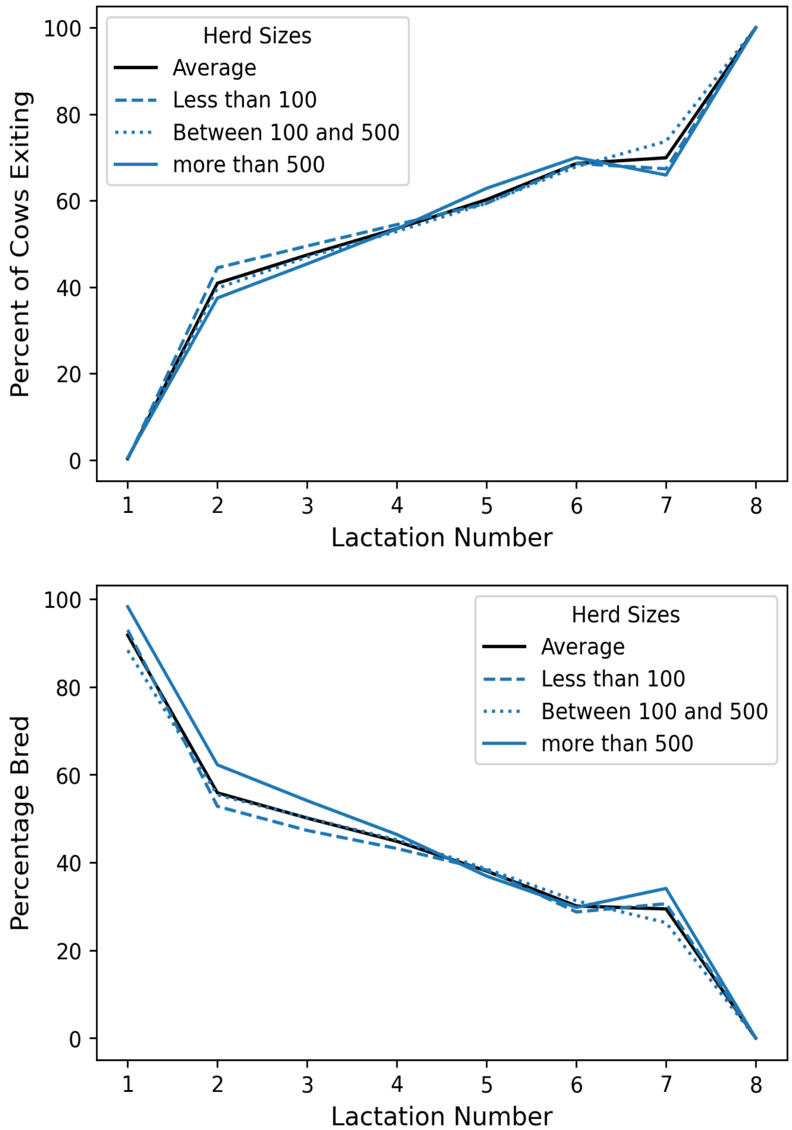


Figure 4: Percent Exiting/Bred at Each Lactation
(Uncensored Cows Only)

first four lactations. Larger farms may have an advantage in the fact that they can afford to inseminate in more cases without concern for cost, whereas smaller farms may have to be more prudent in their breeding decisions.

Herd testing data often does not explicitly note whether a cow exit is *planned* or *unplanned*, so a combination of data on exit rates and breeding decisions is used to determine whether a cow was explicitly *replaced* or *kept*. Figure 5 explains the criteria for using breeding decisions to determine whether a cow was "replaced." First, if the cow is observed in its next production cycle than it considered kept, or $i_{jt} = 0$. If the cow is not observed in its next production cycle, it is considered replaced if two more criteria are met. First, the dairy must not be bred. If the cow is bred but does not appear in the next cycle, the observation is marked missing and is instead considered an unplanned exit.[10] If the dairy cow is not bred, it must also be the case that it is observed in the herd after 120 days and is absent from the data before the end of the sample (and so is not right-censored). The dairy cow must be observed in the first 120 days because it may be the case that the cow died before it was able to be bred, and so does not constitute a replacement. Since most unplanned mortality occurs in the first four months, all exits happening in this time period are used to construct the survival probability S_{jt}.

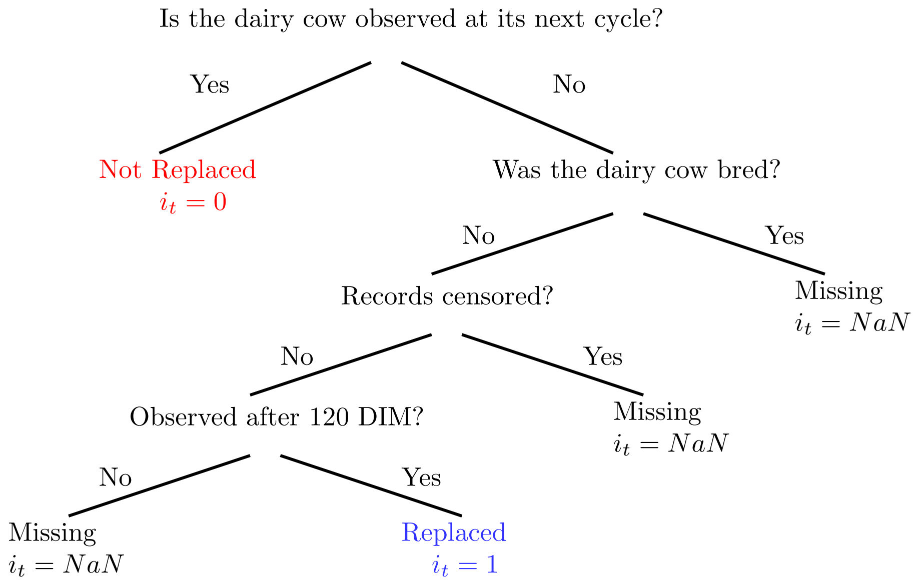
5.2 Survival Rate

To get an estimate of S_{jt} from the data, I estimate the percentage of exits that happen in the first four months of a cow's lactation at each age. Dairy cows are most prone to health shocks during this period, and it is usually not profitable to voluntarily sell an animal in this period.[11] Figure 6 shows exit rates less than or more than 120 days in milk (DIM), which

[10] The costs of breeding are high enough that it is uncommon for a dairy cow to be bred with the intention of being slaughtered. Instead, these cows may be bred for the purposes of selling to another farm, an economic decision that is not the focus of this model. These sorts of decisions are usually categorized as outside of the farm's primary occupation of producing milk (Fetrow et al., 2006).

[11] Dairy cow milk production is the highest in the first four months of production, and selling a cow during before it finishes its cycle is generally not profitable. The costs of sustaining a cow through the calving period and breeding are high enough that dairy farmers aim to keep cows through their whole year long lactation.

Figure 5: Flow Chart for “Breeding” Replacement Definition



is the number of days since the cow has calved. This calculation includes the first lactation cows in order to calculate the probability of unplanned exit for all ages.

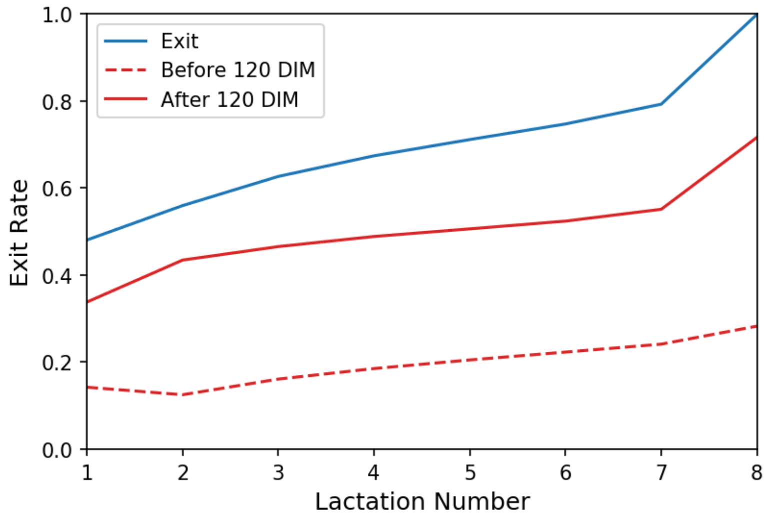
The unplanned exit rate, which is one minus the exit before 120 DIM, has a “bath tub” shape, meaning that the failure rate is higher in the first year than in the second year, but increases after the second year. This is similar to the calculated failure rate for many mechanical components in operations research, and reflects the fact that cows in their first lactation are generally at higher risk for health issues than older cows. The rate estimated in this data is similar to the rate of “involuntary culling” from other studies of unplanned mortality in dairy cows, which suggests it is a good approximation of S_{jt} for this analysis.

5.3 Market Prices and Shocks

In my model, expected revenue R is a function of a “profit-margin” state p_t, replacement cost c_t, and “revenue shock” $\eta_{jt} p_t$. For these two prices, I use income-over-feed-cost (IOFC), a measurement of the profit margin from producing one pound of milk “at test.”[12] This

[12]The IOFC measurement is the return from producing one pound of milk with “average component values” for a given area (in this case Wisconsin). It includes feed cost, labor cost, and capital cost, and generally reflects the “average profitability” of producing one pound of milk. See Gould and Cabrera (2011)

Figure 6: Exit Rate Decomposed and Compared to Literature Involuntary Rates



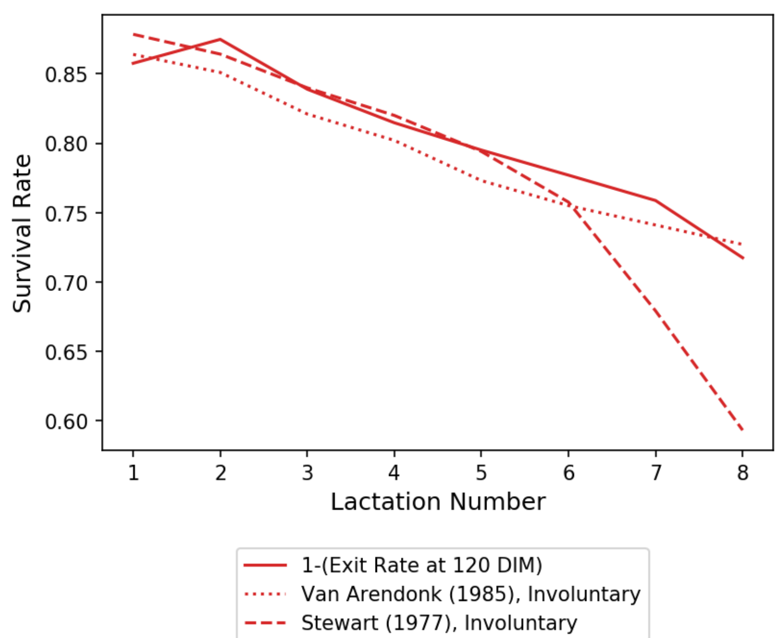
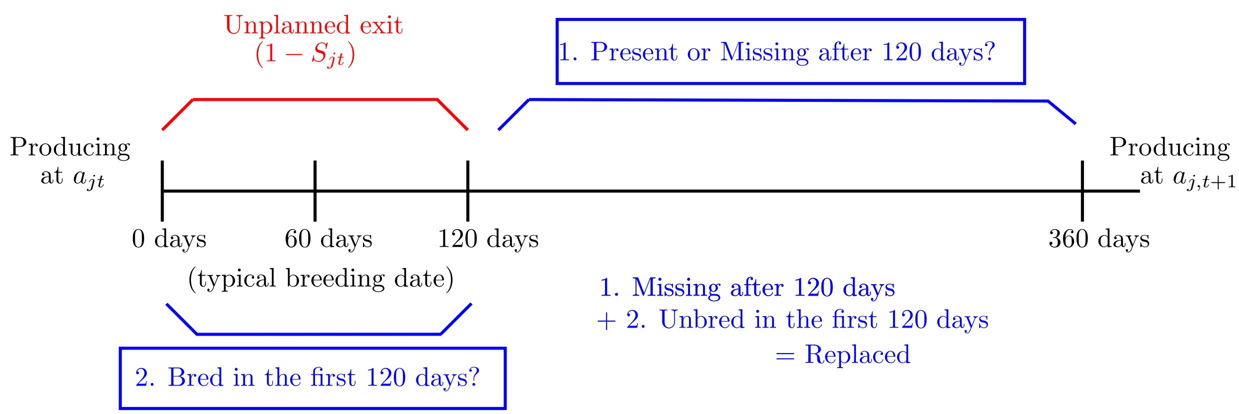


Figure 7

Defining Replacement in the Cow Production Cycle

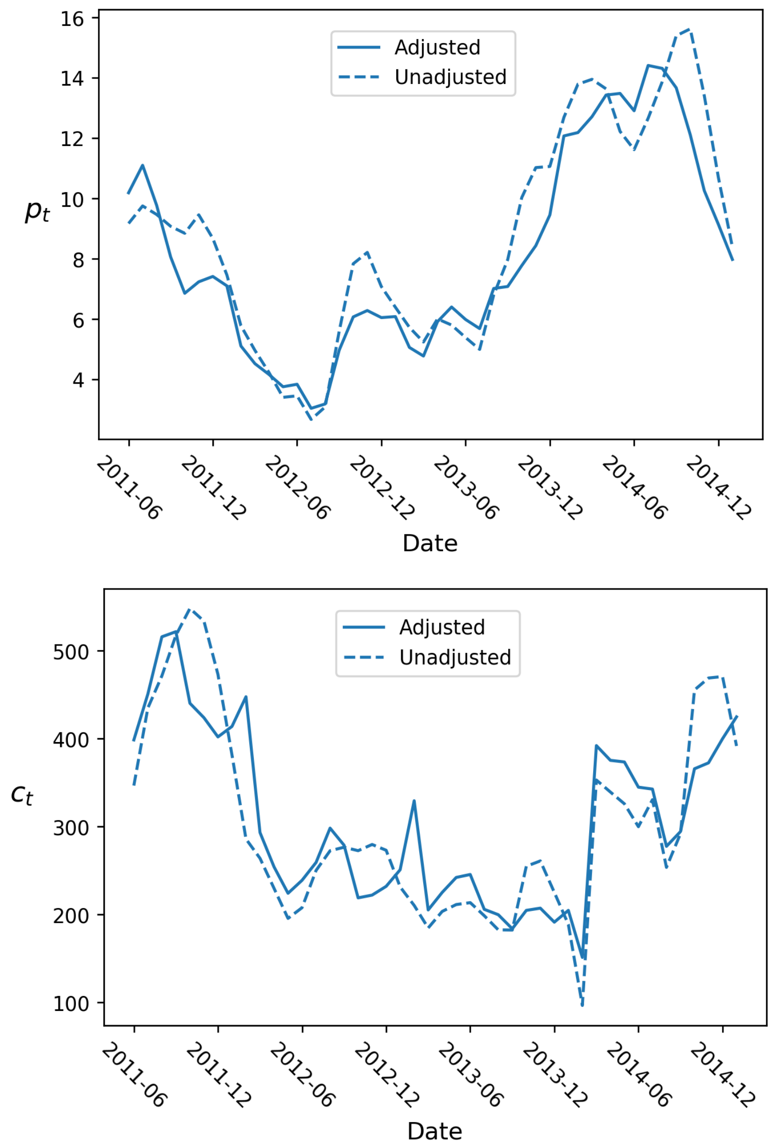


measure of milk profitability is important to producers because it is the one used for USDA price support programs (Gould and Cabrera, 2011). The replacement cost is calculated as the salvage value of a 1,400 pound dairy cow minus the market price for a new heifer. Prices are de-seasonalized, which makes the assumption that dairy farmers seasonally adjust their expectations about price. Because dairy cow replacement is not seasonal in Wisconsin, it is unclear which part of the year dairy farmers observe prices and form expectations. Here I assume adaptive expectation for the expected return R, which means that managers value next period's expected revenue at the most recent prices. To calculate FV, I assume the probability of next period prices to be derived from from an AR(1) regression using the de-seasonalized prices. Figure 8 shows the price trends before and after seasonal correction from the data.

I calculate the production shock η_{jt} from a milk production model, described in Appendix B. The objective of the model is to calculate the performance of the cow relative to its herd mate, taking into account lactation number, lactation length, and milking intensity. The "shock" portion of the production function is calculated as the residual from this regression

for details of calculation.

Figure 8: Price Trends Adjusted and Unadjusted for Seasonality



model. Because the covariate in the model is actually $p_t\eta_{jt}$, I calculate the residual for both fat and protein, multiply them by that lactation's latest Class III component prices, and sum them together. Descriptive statistics of the calculated revenue shocks can be found in Appendix B.

6 Results

Below I estimate the structural model developed in the previous section using the ECCP method. Standard errors in all models are estimated as the standard deviation of 1,000 bootstrap replications. I use regression weights in all calculations to take into account the fact that the accuracy of FV decreases in regions of the state space with few observations. I weigh observations less if the particular combination of states is not seen often in the data. In all models, the discount rate is fixed at .99 unless otherwise specified. The tables present the structural parameters, which are nonlinear combinations of the reduced form parameters. To interpret the coefficients in dollar terms, I divide by the scale λ, which represents the "marginal utility of money" (since the coefficient of $S_{jt}c_t$ is -1 in the model). The estimate of λ is presented in all tables at the bottom. I finish this section by estimating the model on different herd-size categories to explore heterogeneity in unplanned mortality cost across farm types.

6.1 First-Stage Estimation

The first-stage estimation of FV proceeds in three steps. First, the transition probabilities f_x and f_z are calculated from data on prices and shocks. Second, the CCP P_1 is calculated on every combination of states after being trained on the sample. Finally, FV is derived by taking the expectation over P_1 using the derived distribution of p, c, and η. A random forest algorithm is used to prevent over-fitting and assure good out-of-sample properties for the prediction of P_1.

State Transitions I use the following equations to estimate the distributions of p_t, c_t and η_t assuming that the error term is normally distributed:

$$p_t = \mu_p/(1-\rho_p) + \rho_p p_{t-1} + \upsilon^p, \quad \upsilon^p \sim N(0, \sigma_p^2)$$

$$c_t = \mu_c/(1-\rho_c) + \rho_c c_{t-1} + \upsilon^c, \quad \upsilon^c \sim N(0, \sigma_c^2)$$

$$\eta_t = \mu_\eta/(1-\rho_\eta) + \rho_\eta \eta_{t-1} + \upsilon^\eta, \quad \upsilon^\eta \sim N(0, \sigma_\eta^2)$$

The predictions are done using monthly data that was de-seasonalized and CPI adjusted. Table 2 presents the results of these regressions, as well as the results of an AR(1) regression to estimate the initial value of ρ to use in the state transitions for η.

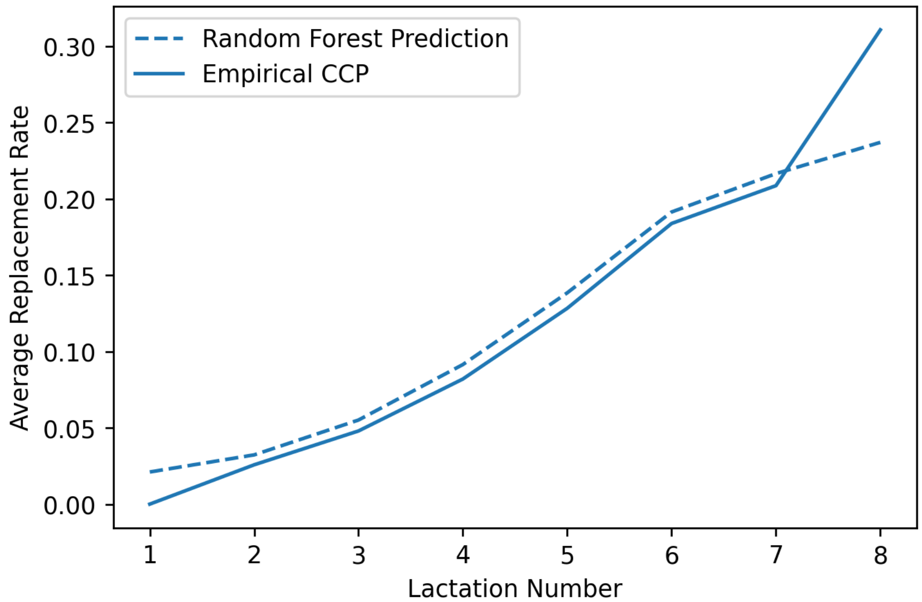
Table 2: AR(1) Regressions for State Transitions

	η	p	c
μ	37.14	7.88	314.87
	(0.001)	(1.41)	(49.46)
σ	758.01	1.01	55.63
	(0.92)	(0.11)	(0.97)
ρ	0.373	0.952	0.830
	(0.002)	(0.047)	(0.125)
No. Obs.	405,467	44	44

Note: standard errors are in parentheses

Conditional Choice Probabilities The random forest algorithm was trained using Brier score loss and 10-fold cross-validation. Figure 9 shows the performance of the random forest estimator in-sample as compared with the empirical probabilities, which are the average replacement rates at each lactation number. The biggest deviation in performance for the random forest is at higher ages given the small number of observations at that level (which comprise less than 1% of the data).

Figure 9: CCP In-Sample Predictions



Brier Score: $\frac{1}{NT}\sum_{t=1}^{T}\sum_{j=1}^{N}(\hat{P}_{jt}^{1} - i_{jt})^2$

	Flexible Logit	Random Forest
Brier Score	0.1818	0.1778

Note: lower Brier Score implies better performance.

The table in Figure 9 also compares the performance of flexible logit versus random forest. When compared with flexible logit, the random forest performs better, but only marginally so. The downside of random forest was its tendency to predict zeros and ones, which causes a problem for estimation (since lnP_1 is one of the covariates). Flexible logit, on the other hand, did a better job of predicting probabilities other than zero and one in-sample, so it was used for all in-sample probabilities. However, in simulations random forest performed the best on out-of-sample estimations of P_1, and so I use random forest estimates for the calculations of FV.

6.2 Second Stage Estimates

I estimate the model without unplanned mortality (the Miranda and Schnitkey (1995) specification), with a zero discount rate (a myopic decision maker), and two different discount rates corresponding to a 5% and 1% interest rate. Table 3 shows the estimates using the definition of replacement based on a combination of breeding and exit. Here the estimate of α is about 2,300 USD per exit. As a benchmark, in the year 2011 the average market price for a dairy heifer replacement was on average 1,400 USD (USDA-NASS, 2011). This cost is also 1,300 USD higher than the upper range of the estimates that De Vries (2013) gives for the cost of "involuntary culling." The marginal cost is about 120 USD per year, which is similar to the estimate of Miranda and Schnitkey (1995). The age at which cows maximize production is in line with the empirical production function calculated in Appendix B, and the production shock correlation ρ is closer to the first stage estimate of 0.37.

The time trend is estimated as negative, which means that over time farmers were more likely to keep their cows. This is not consistent with genetic progress being a motivating factor in animal replacement, the often cited explanation for aggressive replacement policies on dairy farms (De Vries, 2017; Miranda and Schnitkey, 1995; Smith, 1973). Over time, managers are more likely to hold on to their current cows, in direct contradiction with the challenger-versus-defender model of asset replacement and technological progress. This result is robust to using an alternative time trend based on cow birth year, suggesting this is a robust feature of the decision and not a fluke of the data (see Appendix C for more details).

The ECCP model also estimates a distribution of cow-specific fixed effects. These effects are individual "replacement premiums" for each animal and are presented in Figure 10 over age. My hypothesis was that including unplanned mortality would help explain the large, positive intercept for replacement found in Miranda and Schnitkey (1995). When introducing unplanned mortality into the model, the distribution of fixed effects noticeably shifts to the left. This provides evidence that the large and positive intercept in their model can be

Figure 10: Fixed Effects Distribution over Age

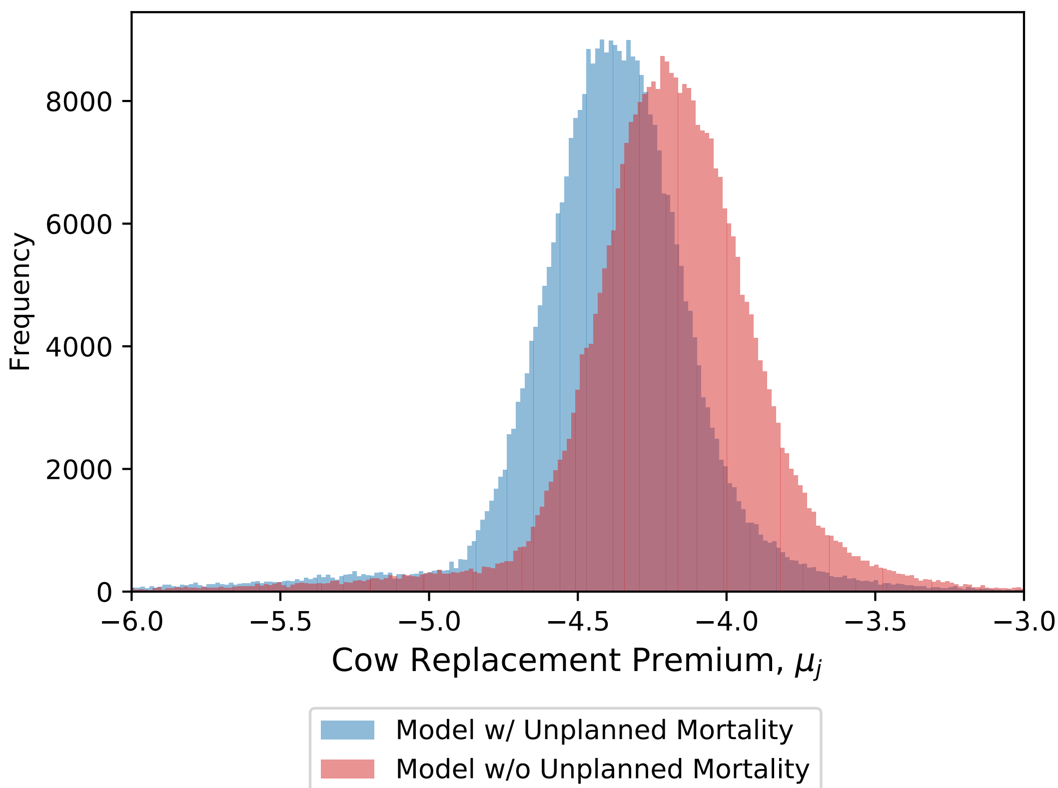


Table 3: Structural Model Results

		No Mortality	Mortality Risk $\delta = 0$	Mortality Risk $\delta = .95$	Mortality Risk $\delta = .99$
Time Trend	τ	-21.98	-12.82	-16.38	-16.64
		(0.86)	(0.40)	(0.69)	(0.71)
Penalty	α		1,911.50	2,330.74	2,371.84
			(70.80)	(116.10)	(115.30)
MC	γ	132.33	299.47	127.35	114.84
		(10.90)	(7.93)	(10.25)	(10.42)
Shock Correlation	ρ	0.405	0.226	0.333	0.341
		(0.0113)	(0.0040)	(0.0087)	(0.0092)
Age of Max	$-\frac{\beta_1}{2\beta_2}$	3.83	2.35	3.55	3.60
		(0.026)	(0.021)	(0.022)	(0.023)
Scale	λ	0.0014	0.0029	0.0017	0.0017
		(0.00004)	(0.00004)	(0.00004)	(0.00004)
Observations		355,734	355,734	355,734	355,734
Adjusted R^2		0.215	0.462	0.242	0.234

Bootstrapped standard deviations in parentheses

explained by the costs of unplanned mortality. These unobserved traits are likely genetic factors that only the manager observes, yet are factors taken into account in replacement decisions. Note that these premiums are independent of observed performance, already captured in the state η. The large variance in effect also shows the importance of controlling for this kind of unobserved heterogeneity when modeling the replacement of assets that are as heterogeneous as dairy animals.

Additional robustness checks are found in in Appendix C, including using different levels of fixed effects, using an alternative time trend, and including somatic cell count as a covariate. Using only herd level fixed effects resulted in a lower estimate of α, indicating that animal-level effects had significant effects on the decision rule and on the estimation of the unplanned mortality cost. When using a time trend based on the year the animal was born instead of the month of observation, the time trend still had a negative effect on the propensity to replace dairy cows. Older cows are more likely to be kept, suggesting that accelerating progress in dairy genetics does not explain high replacement rates. Finally, including somatic cell count as a proxy for health shocks affects the probability of replacement but does not not meaningfully change the parameter estimates.

6.3 Heterogeneity across Herds

To analyze the heterogeneity in unplanned mortality cost across farm type, I estimate the model on different herd-size categories Table 4 shows estimates the model on different categories of herd size to determine whether the model's results change significantly across herd type.

Table 4: Structural Model Estimates Across Farm Type

		Less than 100	100 to 250	250 to 500	500 to 1000	More than 1000
Time Trend	τ	-21.87	-19.93	-18.32	-13.50	-13.92
		(2.10)	(1.91)	(1.65)	(1.60)	(1.51)
Penalty	α	3,795.79	3,616.42	2,062.43	1,367.64	2,152.54
		(394.86)	(359.54)	(258.39)	(286.25)	(220.61)
MC	γ	166.38	134.18	153.340	101.39	62.02
		(30.40)	(27.43)	(24.49)	(24.02)	(20.74)
Shock Correlation	ρ	0.440	0.411	0.364	0.319	0.206
		(0.0347)	(0.0284)	(0.0211)	(0.0216)	(0.0103)
Age of Max	$-\frac{\beta_1}{2\beta_2}$	3.99	3.82	3.45	3.38	3.45
		(0.063)	(0.057)	(0.048)	(0.055)	(0.044)
Scale	λ	0.0011	0.0013	0.0016	0.0018	0.0028
		(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Observations		74,161	85,127	86,684	59,135	50,627
Adjusted R^2		0.178	0.206	0.269	0.260	0.223

Bootstrapped standard deviations in parentheses
Discount rate set to .99

Farms with less than 250 dairy cows perceive an unplanned mortality cost nearly three times higher than farms with between 500 and 1000 cows: 3,800 USD per exit versus 1,360 USD per exit. Farms with between 250 and 500 dairy cows pay about 3,600 whereas the largest dairy farms, which make up a very small percentage of herds in this data, pay about 2,100 USD. Taking 1,400 USD as an average market rate for a dairy cow, only farms with 500 to 1000 cows perceive the cost to be about the market rate of a replacement heifer. There are a few possible explanations for this discrepancy across herd size. One, large farms may benefit from economies of scale in dealing with the costs from unplanned exits. This cost savings can be from disposing of animal carcasses more efficiently or having lower costs of treating diseases. The cost may also be lower if large farms are able to keep more replacements on hand, which lowers the search cost when a cow unexpectedly leaves the herd or even reduces the amount of time that a stall is empty. In contrast, small farms may have less replacements on hand, meaning a slot cannot be filled as quickly because replacements are harder to find. Regardless of the source of the cost, small dairies in this sample pay disproportionately more per death than large dairies, which suggests that reductions in animal health in favor of production may disproportionately affect the profitability of small dairies.

7 Welfare Analysis

The parameter α represents the added cost to replacement induced by an unplanned mortality event. As a counterfactual exercise, we can examine the price farmers would pay to eliminate mortality risk completely. Earlier, I showed how including S_{jt} distinguishes this model from that of Miranda and Schnitkey (1995). The payoff with and without the probability of unplanned mortality is:

$$\begin{aligned}\theta^0 X^0_{jt} &= \mu + \tau t + \alpha(1 - S_{jt}) - \rho\eta_{jt}p_t - S_{jt}c_t + \gamma S_{jt}a_{jt} \\ &\quad - (\beta_1 + 2\beta_2)S_{jt}a_{jt}p_t - \beta_2 S_{jt}ta^2_{jt}p_t + \delta\Delta V^1_{jt} + \delta\Delta V^2_{jt}S_{jt} \\ \theta^1 X^1_{jt} &= \mu + \tau t + \rho\eta_{jt}p_t - c_t + \gamma a_{jt} \\ &\quad - (\beta_1 + 2\beta_2)a_{jt}p_t - \beta_2 a^2_{jt}p_t + \delta\Delta V^1_{jt} + \delta\Delta V^2_{jt}\end{aligned}$$

Since $\theta^1 X^1_{jt}$ is the payoff a manager receives when every cow will survive to the next lactation if it is not replaced ($S_{jt} = 1 \quad \forall \quad a_{jt}$), the willingness-to-pay to eliminate unplanned mortality is the amount of money that would need to be given to a farmer to make them indifferent between a payoff function with health risk ($\theta^0 X^0_{jt}$) and a payoff without health risk ($\theta^1 X^1_{jt}$). This is a more comprehensive estimate of mortality cost than α because it takes into account the effects of S_{jt} on the entire payoff function. This number is analogous to the premium of an insurance policy for unplanned mortality.

Assuming that λ is the same for every farmer, the average compensating variation for transitioning to this payoff function is:

$$E(CV(X^1, X^0, \theta^1, \theta^0)) = \frac{1}{\lambda}\Big(ln(1 + e^{-\theta^1 X^1_{jt}}) - ln(1 + e^{-\theta^0 X^0_{jt}})\Big),$$

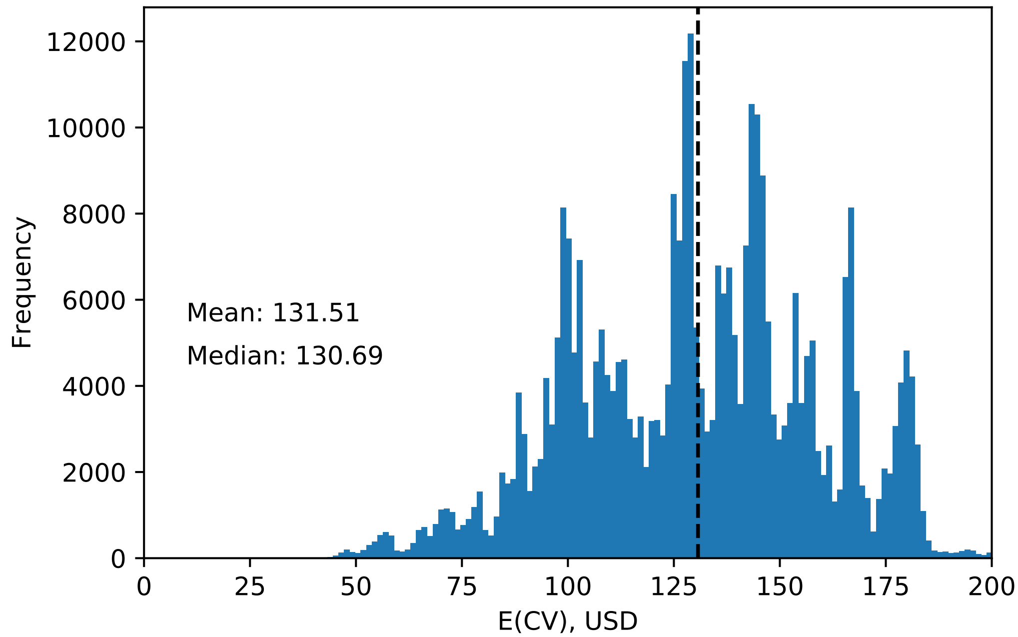
which is derived in Small and Rosen (1981).[13] The relationship between this compensating variation measure and α can be described by this partial derivative:

$$\frac{\partial E(CV)}{\partial \alpha} = (1 - S_{jt})P_0(x_{jt}, z_t).$$

The extent to which compensating variation increases with increasing in α is determined by the probability of the event $(1-S_{jt})$ and the probability the farmer wants to keep the cow ($P_0(x_{jt}, z_t)$. If the cow were definitely going to die ($S_{jt} = 0$), and the manager were definitely going to keep the cow ($P_0 = 1$), then a one-dollar increase in α increases compensating variation by one dollar. Anything that increases the probability of keeping the animal will also increase $E(CV)$. $E(CV)$ can be thought of as the premium for an insurance policy for animal mortality, and dividing this by the probability of the event $(1 - S_{jt})$ gives an expected "indemnity" for unplanned mortality that is more comprehensive than the penalty parameter α. Because the ECCP method recovers estimates of the cow-specific intercepts, the fixed effects can be incorporated in the calculation of $E(CV)$ (unlike conditional logit). This captures the ability of managers to see characteristics about cows that are unobserved in the model, giving us a sense for their importance in determining how managers value

[13] Recall also that I fixed the payoff to replacement to be zero having subtracted the payoff from replacement from both payoffs, so $-\theta^1 X^1_{jt}$ is the payoff from keeping the animal

Figure 11: Compensating Variation



unplanned mortality.

In Figure 11 I graph the expected CV over the whole sample. On average, managers would insure their cows for 130.26 USD per lactation. First lactation cows have the highest CV, which follows as managers would pay more to insure dairy cows that have more producing potential. The implied indemnity of an unplanned death $(E(CV)/(1 - S_{jt}))$ for a new cow is about 1,900 USD. Unobserved cow characteristics influence the willingness to pay to eliminate risk significantly. A subset of older cows in particular elicits a high $E(CV)$, since cows kept this long have unobserved traits that prompt them to be kept longer. Broken out by herd size, the same pattern emerges as in Table 3. Farms with less than 250 cows would pay the most to eliminate mortality, almost three times more than farms with between 500 and 1000.

Table 5: Average Indemnity by Age

	Mean	Std Dev	Count	Percentiles 2.5%	50%	97.5%
Age						
1	1975.13	171.27	100,090	1705.29	1952.98	2,248.56
2	1484.27	196.79	127,813	1,163.14	1,529.68	1,826.88
3	1062.27	238.25	70,437	693.44	1,094.19	1,478.75
4	767.35	220.89	34,127	409.82	745.15	1,134.47
5	693.97	306.54	14,877	293.33	664.83	1,386.93
6	772.06	339.43	5,826	359.66	681.58	1,552.97
7	891.17	410.40	1,988	383.10	795.12	1,863.20
8	1227.35	636.85	576.	481.72	1,119.06	2,675.44

Table 6: Average Indemnity for New Cows, by Herd Size

	Mean	Std Dev	2.5%	50%	97.5%
Herd Size					
Less than 100	2,434.78	567.57	1,267.55	2,487.20	3,408.60
100 to 250	2,640.18	457.15	1,884.86	2,659.11	3,353.25
250 to 500	1,160.75	456.91	161.80	1,193.53	1,892.32
500 to 1000	657.35	418.90	-253.26	694.28	1,305.05
more than 1000	1,977.15	556.78	1,469.33	1,866.96	3,458.29

8 Discussion and Conclusion

Using a structural dynamic discrete choice model, I examine asset replacement behavior on Wisconsin dairy farms and the effect of asset failure and depreciation on dairy cow replacement. Replacing dairy cows is unique in asset replacement since new replacements have to be procured ten months ahead of time, meaning premature dairy cow exit can cause significant costs to the operation. I test the hypothesis that costs arising from this "unplanned mortality" can explain why dairy farms replace cows before it is considered optimal from the perspective of most models. I derive a dynamic discrete choice model that explicitly incorporates the probability of unplanned mortality as a replacement motive and backs out a cost parameter representing profit losses from unplanned mortality due to declining animal health. The analysis finds that the cost of unplanned mortality is 2,300 USD per exit, and is even larger on small dairies. I used the estimates of these fixed effects to calculate expected compensating variation that takes into account unobserved cow variation, which finds that dairy farmers would, on average, pay 1,900 USD to eliminate mortality risk completely. My results suggest that part of the discrepancy between optimal culling rules calculated from simulations and observed replacement decisions is explained by the fact that simulations underestimate the cost of unplanned mortality. In addition to shedding light on this empirical puzzle, the results have significant implications for the debate around dairy sector profitability and the role of animal health.

First, this analysis finds large disparities in mortality cost across herd size. Farms under 250 dairy cows pay roughly three times more per death, and these dairies are also willing to pay nearly three times more to eliminate mortality risk completely. These results suggest that small dairy farms may be disproportionately paying the cost of the current trends in animal health. Since the costs of unplanned mortality are likely related to the capacity to hold replacements on hand, other technologies may be limited in their ability to bring this cost down. Milk production has been prioritized in breeding to increase the profitability of dairy, but if smaller farms pay more of the downside then this breeding strategy could have unforeseen distributional effects. In fact, such a breeding strategy may inadvertently contribute to the increased consolidation of the dairy industry if it causes small farms to exit at higher rates (Shepel, 2019).

A second policy implication is that improving animal health at the cost of production could actually increase profitability of dairy in some cases. In fact, this analysis demonstrates that improving animal health grants even more cost savings than previously assumed (De Vries, 2013). This suggests that there is, as De Vries (2017) argues, policy rationale for investing in a technology that balances health and production. Increasing cow life-span also has the potential to address the negative environmental externalities caused by short cow life and also improve animal welfare (Garnsworthy, 2004; Oltenacu and Broom, 2010).

By using actual replacement decisions, my research calibrates these costs not by simulations but by the actual behavior of dairy farmers. In the language of Rust (1987), this is an example of a "bottom-up" approach to understanding some of the aggregate trends in an industry like dairy. Thanks to advances in dynamic discrete choice, this analysis can be done more efficiently and more rigorously, controlling for a broader ranger of heterogeneity than possible before. Advancements in machine learning also enhance dynamic discrete choice estimation by simplifying and improving the estimation of first-stage probabilities over state spaces that are fully continuous. Thanks to such advancements, these kinds of models can produce insights into firm behavior and perspectives on policy at a level of empirical rigour never before possible.

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Appendix A Future Value Calculation

$$v(x_{jt},1) - v(x_{jt},0) = R(x_{jt},1) - R(x_{jt},0) + \delta \sum_{x_{t+1}=1}^{X} \Big(R(x_{t+1},1) + ln(P_1(x_{t+1}),1) \Big) \Big(f(x_{t+1}|x_{jt},1) - f(x_{t+1}|x_{jt},0) \Big)$$

The last term multiplied by δ is what I call ΔV, and I derive it below. Remembering that all the states evolve independently of one another and only a_t and η_t depend on i_t, we can factor the probabilities out this way:

$$f(x_{t+1}|x_{jt},1) - f(x_{t+1}|x_{jt},0) = \Big(f(a_{t+1}|a_t,1) f(\eta_{t+1}|\eta_t,1) - f(a_{t+1}|a_t,0) f(\eta_{t+1}|\eta_t,0) \Big) f(z_{t+1}|z_t)$$

When considering a_t, recall that a_t is a discrete state that can only transition to $a_{t+1} = 1$ or $a_{t+1} = a_t + 1$; the age must either go up by one or go back to 1, so it sufficient to only consider the cases where $a_{t+1} = a_t + 1$ or $a_{t+1} = 1$ when calculating the expected value. Because of unplanned exit, the probability of transitioning from age a_t back to age 1 is:

$$f(a_{t+1} = 1|a_t, i_t) = \begin{cases} 1 & i_t = 1 \\ 1 - S(a_t) & i_t = 0 \end{cases}$$

And that the probability of a_t going to age $a_t + 1$ is:

$$f(a_{t+1} = a_t + 1|a_t, i_t) = \begin{cases} 0 & i_t = 1 \\ S(a_t) & i_t = 0 \end{cases}$$

The shock state η_t is also dependent on the decision to replace. Recall that shocks are auto-correlated with coefficient ρ but only in the case that the cow is not replaced; should the cow be replaced, the performance of the previous cycle does not affect the new occupant.

Now I calculate the difference in transition probabilities for $a_{t+1} = a_t + 1$ and $a_{t+1} = 1$.

$$\begin{aligned} f(1, \eta_{t+1} | a_t, \eta_t, 1) - f(1, \eta_{t+1} | a_t, \eta_t, 0) &= (1) f(\eta_{t+1} | \eta_t, 1) - (1 - S(a_t)) f(\eta_{t+1} | \eta_t, 0) \\ &= f(\eta_{t+1} | \eta_t, 1) - (1 - S(a_t)) f(\eta_{t+1} | \eta_t, 0) \end{aligned}$$

$$\begin{aligned} f(a_t + 1, \eta_{t+1} | a_t, \eta_t, 1) - f(a_t + 1, \eta_{t+1} | a_t, \eta_t, 0) &= (0) f(\eta_{t+1} | \eta_t, 1) - S(a_t) f(\eta_{t+1} | \eta_t, 0)) \\ &= -S(a_t) f(\eta_{t+1} | \eta_t, 0)) \end{aligned}$$

Now back to calculation of ΔV, first ignoring the states p_t and c_t since they are not influenced by the decision:

$$
\begin{aligned}
\Delta V =& \sum_{x_{t+1}=1}^{X} \Big(R(x_{t+1},1) + ln(P_1(x_{t+1})\Big)\Big(f(x_{t+1}|x_{jt},1) - f(x_{t+1}|x_{jt},0)\Big) \\
=& \sum_{\eta_{t+1}}^{E} \Big(R(1,\eta_{t+1},z_{t+1},1) + ln(P_1(1,\eta_{t+1},z_{t+1})\Big) \\
& \qquad \Big(f(\eta_{t+1}|\eta_t,1) - (1-S(a_t))f(\eta_{t+1}|\eta_t,0)\Big) - \\
& \sum_{\eta_{t+1}}^{E} \Big(R(a_t+1,\eta_{t+1},z_{t+1},1) + ln(P_1(a_t+1,\eta_{t+1},z_{t+1})\Big)\Big(S(a_t)f(\eta_{t+1}|\eta_t,0))\Big) \\
=& - S(a_t)\sum_{\eta_{t+1}}^{E} \Big(R(a_t+1,\eta_{t+1},z_{t+1},1) + ln(P_1(a_t+1,\eta_{t+1},z_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,0)\Big) \\
& + \sum_{\eta_{t+1}}^{E} \Big(R(1,\eta_{t+1},z_{t+1},1) + ln(P_1(1,\eta_{t+1},z_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,1)\Big) \\
& - \Big(1-S(a_t)\Big)\sum_{\eta_{t+1}}^{E} \Big(R(1,\eta_{t+1},z_{t+1},1) + ln(P_1(1,\eta_{t+1},z_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,0)\Big) \\
=& S(a_t)\sum_{\eta_{t+1}}^{E} \Big(ln(P_1(1,\eta_{t+1},z_{t+1}) - ln(P_1(a_t+1,\eta_{t+1},z_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,1)\Big) \\
& + \sum_{\eta_{t+1}}^{E} \Big(R(a_{t+1}=1,\eta_{t+1},z_{t+1},1) + ln(P_1(1,\eta_{t+1},z_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,1) - f(\eta_{t+1}|\eta_t,0)\Big)
\end{aligned}
$$

Applying the normalization $R(x_{jt},1) = 0$ and using the shorthand $P_1(a_t,\eta_t,p_t,c_t) = P_1(a_t,\tilde{x}_t)$, I can write:

$$
\begin{aligned}
\Delta V =& S(a_t)\sum_{z_{t+1}=1}^{Z}\sum_{\eta_{t+1}=1}^{E} \Big(lnP_1(1,\tilde{x}_{t+1}) - lnP_1(a_t+1,\tilde{x}_{t+1})\Big)f_\eta(\eta_{t+1}|\eta_t,1)f_z(z_{t+1}|z_t) \\
& + \sum_{z_{t+1}=1}^{Z}\sum_{\eta_{t+1}=1}^{E} \Big(ln(P_1(1,\tilde{x}_{t+1})\Big)\Big(f(\eta_{t+1}|\eta_t,1) - f_\eta(\eta_{t+1}|\eta_t,0)\Big)f_z(z_{t+1}|z_t)
\end{aligned}
$$

So now I have factored out the survival function $S(a_t)$ so that I only can estimate its parameters inside the main model. The other state transitions, however, still have to be estimated separately. Note that the value FV_1 has to do with the fact that shocks are correlated, since when $\rho = 0$ then $f(\eta_{t+1}|\eta_t, 1) = f(\eta_{t+1}|\eta_t, 0)$ and $FV_1 = 0$, whereas FV_2 is an adjustment term for the change in the probability of replacing next period if replacement is done today.

Appendix B Milk Production Model

One of the covariates in our model is $\eta_{jt} p_t$, the shock in revenue from the current cycle. To get an estimate of η_{jt}, the deviation from the production function, I do a linear prediction of fat and protein yield for each cow using their covariates. The covariates W_{jt} come from similar models estimated in animal science production models on DHI data (see Kearney et al. (2004) as an example).

The prediction model:

$$y_{jt} = \beta W_{jt} + h_j + \eta_{jt}$$

Contained in W_{jt}:

- Lactation number
- Lactation number squared
- Proportion Days Milked 3x
- Lactation Length (DIM)
- Calving Month
- Birth Year
- Age at first calving

and h_j is a herd intercept. I then predict the residual $\hat{\eta}_{jt} = y_{jt} - \hat{\beta} W_{jt} - \hat{h}_j$ for fat and protein and multiply them by their Class III component prices prevailing in the month the record was taken.

Table 7 shows the results of the milk production model. Calculated from this production function, the optimal lactation number at which production is maximized is around three to four, which is in line with Miranda and Schnitkey (1995). This indicates that another reason dairy cows are replaced earlier than typically calculated by simulations is that production is maximized much sooner than five lactations. The birth year effects show something akin to genetic progress in milk production; independent of all factors, cows that were born in more recent years have higher milk production.

Figure 12 shows the calculated revenue shock over age. As can be seen in the confidence intervals, they are highly variable and are not statistically different than zero for any age. The variation in this variable is high, reflected in the fact that the standard deviation is about 600 USD. The correlation between shocks is 0.37, implying a mild autocorrelation between lactations.

Appendix C Robustness Checks

C.1 Different Levels of Fixed Effects

Below I estimate the ECCP model with different levels of fixed effects. Using no fixed effects, the cost of unplanned mortality is significantly lower: about 600 USD per exit as opposed to 2,300 USD. The estimate is almost unchanged when using herd fixed effects, which suggests herd level heterogeneity does not have a large effect on the estimation of unplanned mortality cost. In these specifications, the maintenance cost γ is twice as large as in the main specification, which could mean that without animal-level effects the costs are attributed to unit increases in age rather than the identity of the cow. When including animal-level intercepts, the results change significantly: the shock correlation is lower, and the cost of unplanned mortality is almost four times higher. Since the shock correlation appears to be too high without animal-level effects, the animal-level effects model appears to fit the data better (even though the R-squared is lower).

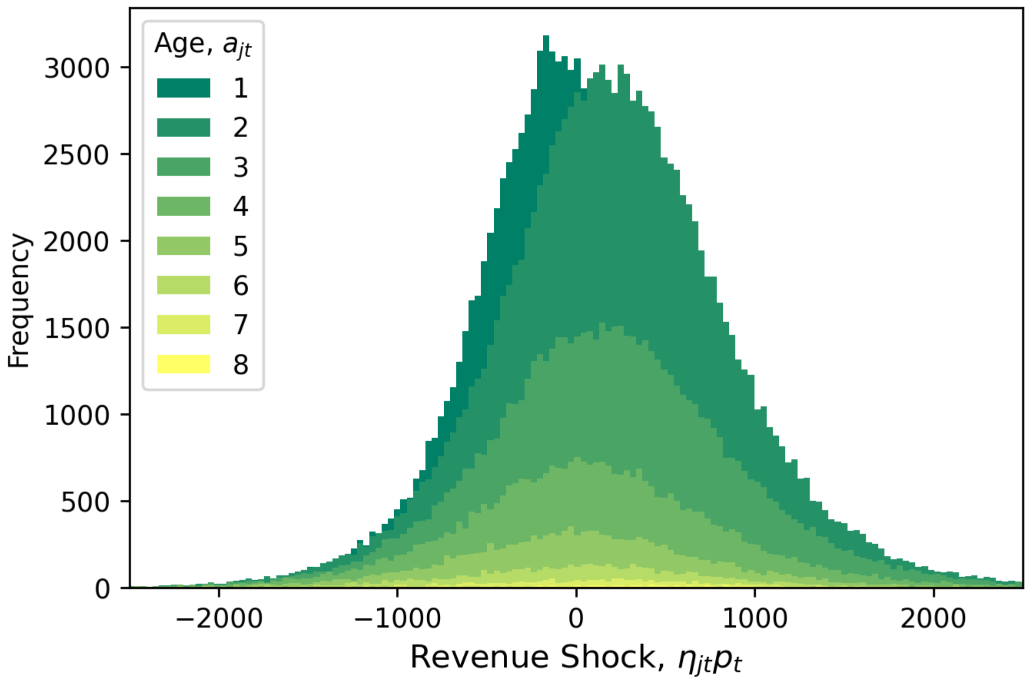

Figure 12: Revenue Shock by Age

	Mean	Std Dev	Count	Percentiles 2.5%	50%	97.5%
Age						
1	-110.75	584.87	322,842	-1,297.59	-105.55	1,040.42
2	136.05	676.23	241,250	-1,171.22	120.21	1,538.50
3	64.81	730.42	153,147	-1,307.91	40.48	1,589.48
4	-40.32	746.20	83,689	-1,445.45	-60.69	1,522.15
5	-80.61	724.43	39,950	-1,477.22	-86.83	1,394.08
6	-74.21	713.33	16,718	-1,498.12	-61.58	1,309.33
7	-19.51	725.10	6,217	-1,547.24	4.17	1,352.78
8	85.86	741.36	2,198	-1,490.17	132.89	1,442.50

Table 7: Milk Production Model

	Fat Yield	Protein Yield	Energy Corrected Milk (ECM)
Lactation Number	66.852***	69.778***	2110.466***
	(0.631)	(0.444)	(15.373)
Lactation Number Squared	-10.973***	-9.686***	-321.745***
	(0.057)	(0.040)	(1.401)
Proportion Milked 3x	99.577***	76.034***	2860.178***
	(1.456)	(1.024)	(35.489)
Lactation Length	3.051***	2.573***	86.508***
	(0.001)	(0.001)	(0.033)
Age in Years	53.974***	33.396***	1358.848***
	(0.461)	(0.324)	(11.239)
Somatic Cell Score	-0.121***	-0.060***	-3.009***
	(0.001)	(0.001)	(0.026)
Birth Year			
2006	34.405***	30.421***	945.791***
	(0.984)	(0.692)	(23.993)
2007	65.036***	57.995***	1754.510***
	(0.954)	(0.671)	(23.255)
2008	75.858***	67.684***	2037.064***
	(0.969)	(0.681)	(23.612)
2009	89.866***	78.058***	2363.118***
	(0.999)	(0.703)	(24.356)
2010	109.260***	91.072***	2807.903***
	(1.037)	(0.730)	(25.284)
2011	110.927***	94.334***	2862.303***
	(1.101)	(0.775)	(26.851)
2012	118.899***	101.299***	3101.160***
	(1.238)	(0.871)	(30.183)
2013	101.899***	90.072***	2510.211***
	(3.035)	(2.135)	(73.983)
Observations	1,172,293	1,172,293	1,172,293
Adjusted R^2	0.86	0.90	0.89

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

These results show that controlling for these asset-level characteristics is important when studying decisions like replacement. Were I to have used a model more in line with previous studies, I would have only controlled only for herd level characteristics and would have understated the cost of unplanned mortality by a significant amount. Including only firm-level effects makes the implicit assumption that the permanent characteristics that affect behavior are constant within a firm, which is not the case here. A dairy herd is a diverse portfolio of genetic types, and in this data an individual cow's characteristics have a large effect on how parameters are estimated. Future economics studies examining management of genetic technology such as animals or crop plots should take these sorts of factors into account when estimating parameters from such data.

C.2 Other Checks

In this section I implement two extra robustness checks: including SCC as a covariate and using an alternative time trend based on the year different cattle were born.

As mentioned previously, unobserved health shocks are a potential endogeneity issue for estimating the coefficient on the hazard rate. Specifically, health shocks may affect replacement and also update the probability of survival. The effects of health states on productivity are already captured in the state η_{jt}, which uses somatic cell count (SCC), a measure of milk bacteria count, in the production function. SCC is an important trait of milk because high counts of SCC are indicative of mastitis, the most prevalent disease among lactating dairy cattle. However, any effect of health states on replacement independent of production is not captured in the model. To test the effect of SCC in the model, I explicitly include it as a covariate. Note that this is making a very specific assumption about how SCC affects replacement; by including it only as as covariate and not as a state, I am assuming it affects next period's payoff but not the continuation value. In Table 9, the coefficient on SCC in the regression is positive, meaning cows with higher bacteria counts are replaced more often, but the coefficient is quite small. A one standard deviation change in SCC,

Table 8: Different Fixed Effects Specifications

		No Fixed Effects	Herd Fixed Effects	Cow Fixed Effects
Premium	μ	-4799.08		
		(152.20)		
Time Trend	τ	-26.75	-26.63	-16.64
		(0.55)	(0.53)	(0.71)
Penalty	α	587.566	651.007	2371.84
		(147.25)	(145.00)	(115.30)
MC	γ	270.12	265.98	114.84
		(10.10)	(10.13)	(10.42)
Shock	ρ	0.58	0.57	0.34
Correlation		(0.02)	(0.02)	(0.01)
Age of Max	$-\frac{\beta_1}{2\beta_2}$	3.76	3.76	3.60
		(0.04)	(0.03)	(0.02)
Age of Free	$-\frac{\beta_1+\beta_2}{\beta_1}$	6.51	6.52	6.20
		(0.07)	(0.07)	(0.05)
Scale	λ	0.0009	0.0009	0.0017
		(0.00005)	(0.00005)	(0.00004)
Observations		355,734	355,734	355,734
Adjusted R^2		0.316	0.318	0.234

Bootstrapped standard deviations in parentheses

Discount rate set to .99

which is 150, implies a change in expected profit of just three dollars. In addition, it does not significantly change the estimate of α.

Another robustness check I implement is using an alternative time trend. In the main specification, I used a simple monthly time trend to capture technological improvement in replacements. My hypothesis was that the time trend would be positively related to replacement since replacing now means taking advantage of new genetics. This is implied by the challenger-versus-defender model of asset replacement, and is theorized in Miranda and Schnitkey (1995) as the cause of the positive culling premium. In all specifications, I find that the time trend is negative, the opposite of what was hypothesized. Rather than being more willing to give up their cattle as time progresses, producers are less willing. The monthly time trend, however, may be picking up another economic condition unrelated to the technology. To test whether this negative time trend really has to do with the technology itself, I use the birth year of the cow as an alternative measure. Instead of a one-unit increase being one month, now a one-unit increase is one year, for example a cow born in 2009 versus a cow born in 2010. The relationship is still limited to be linear, meaning the difference between a 2011 cow and a 2012 cow must be the same between a 2009 and 2010 cow, but the variation is now cow specific rather than time specific. In other words, it directly tests whether a cow being born a year later is replaced more often.

The effect of the time trend remains negative and have an even larger magnitude than the monthly trend. This confirms that the negative time trend has to do with the technology itself. The milk production model in Table 7 shows there are higher returns in milk production associated with increases in this variable, but producers from 2011 to 2014 wanted to hold on to newer cows rather than replace them. The trend does not fit with the theory that genetic progress is an increase in opportunity cost incentivizing dairy farmers to replace earlier (De Vries, 2017). It instead suggests that newer dairy cows are being kept *more*, and they do not expect the trend to continue.

From this robustness check, I conclude that the negative time trend is not a fluke of the data, but actually a robust result of the behavioral model in this data. Dairy farmers' expectations of technological progress do not fit the typical challenger-versus-defender model theorized by Miranda and Schnitkey (1995) and De Vries (2017). Future work should investigate the robustness of this result over longer time spans, as one limitation of this data is that it only covers from 2011 to 2014. These results may be showing short-run expectations about technological process and do not pick up long-run trends that could be explored with a longer panel.

Table 9: Other Robustness Checks

		Main Specification	SCC Shock	Birth Year Time Trend
SCC Shock			-0.0016	
			(0.0123)	
Time Trend	τ	-16.64	-16.66	-180.52
		(0.706)	(0.708)	(17.53)
Penalty	α	2371.84	2372.36	2083.52
		(115.30)	(119.24)	(136.35)
MC	γ	114.84	115.16	-100.63
		(10.42)	(10.40)	(3.60)
Shock Correlation	ρ	0.34	0.34	0.41
		(0.009)	(0.009)	(0.012)
Age of Max	$-\frac{\beta_1}{2\beta_2}$	3.60	3.60	3.43
		(0.023)	(0.023)	(0.022)
Age of Free	$-\frac{\beta_1+\beta_2}{\beta_1}$	6.20	6.20	5.86
		(0.046)	(0.046)	(0.044)
Scale	λ	0.0017	0.0017	0.0014
		(0.00004)	(0.00004)	(0.00004)
Observations		355,734	355,734	355,734
Adjusted R^2		0.234	0.230	0.234

Bootstrapped standard deviations in parentheses

Discount rate set to .99